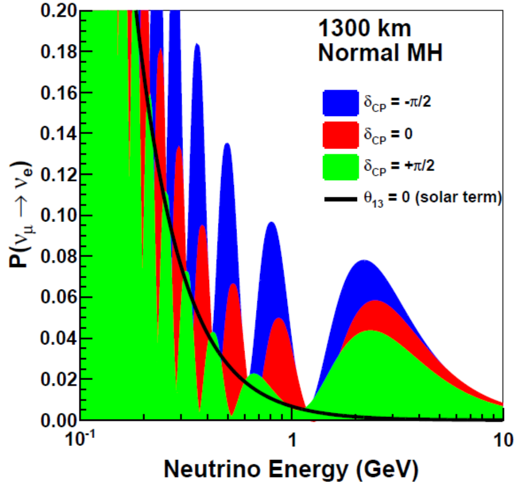
Towards accurate neutrino cross sections: the argon and titanium spectral functions from (*e,e'p*) data

Artur M. Ankowski
University of Wrocław

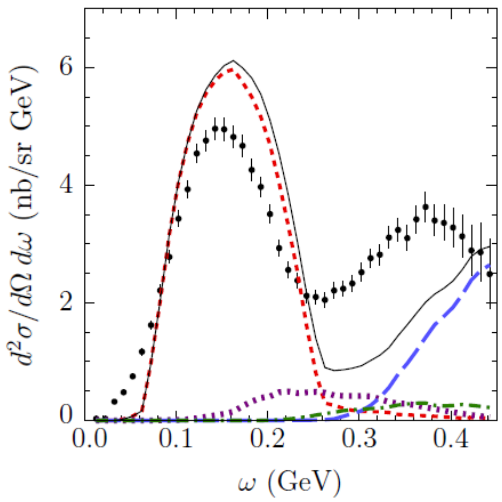
based on L. Jiang *et al.*,
PRD 105, 112002 (2022); PRD 107, 012005 (2023)

PAC Webinar at Nanjing Normal University, May 3, 2023

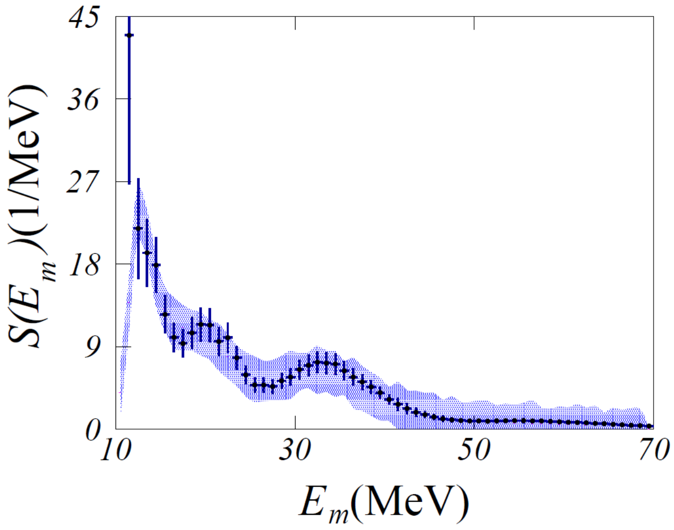
Outline



DUNE Collaboration,
arXiv:1512.06148



A.M.A. & A. Friedland,
PRD 102, 053001 (2020)



L. Jiang *et al.*,
PRD 105, 112002 (2022)

L. Jiang *et al.*,
PRD 107, 012005 (2023)

The “Neutrino” by Bethe and Peierls

“The view has recently been put forward[1] that a neutral particle of about electronic mass, and spin $\frac{1}{2}\hbar$ (where $\hbar=h/2\pi$) exists, and that this ‘neutrino’ is emitted together with an electron in β-decay. This assumption allows the conservation laws for energy and angular momentum to hold in nuclear physics.”

“For an energy 2.3×10^6 volts, … $\sigma < 10^{-44}$ cm^2 … With increasing energy σ increases … but even if one assumes a very steep increase, it seems highly improbable that, even for cosmic ray energies, σ becomes large enough to allow the process to be observed.”

“… **one can conclude that there is no practically possible way of observing the neutrino**”

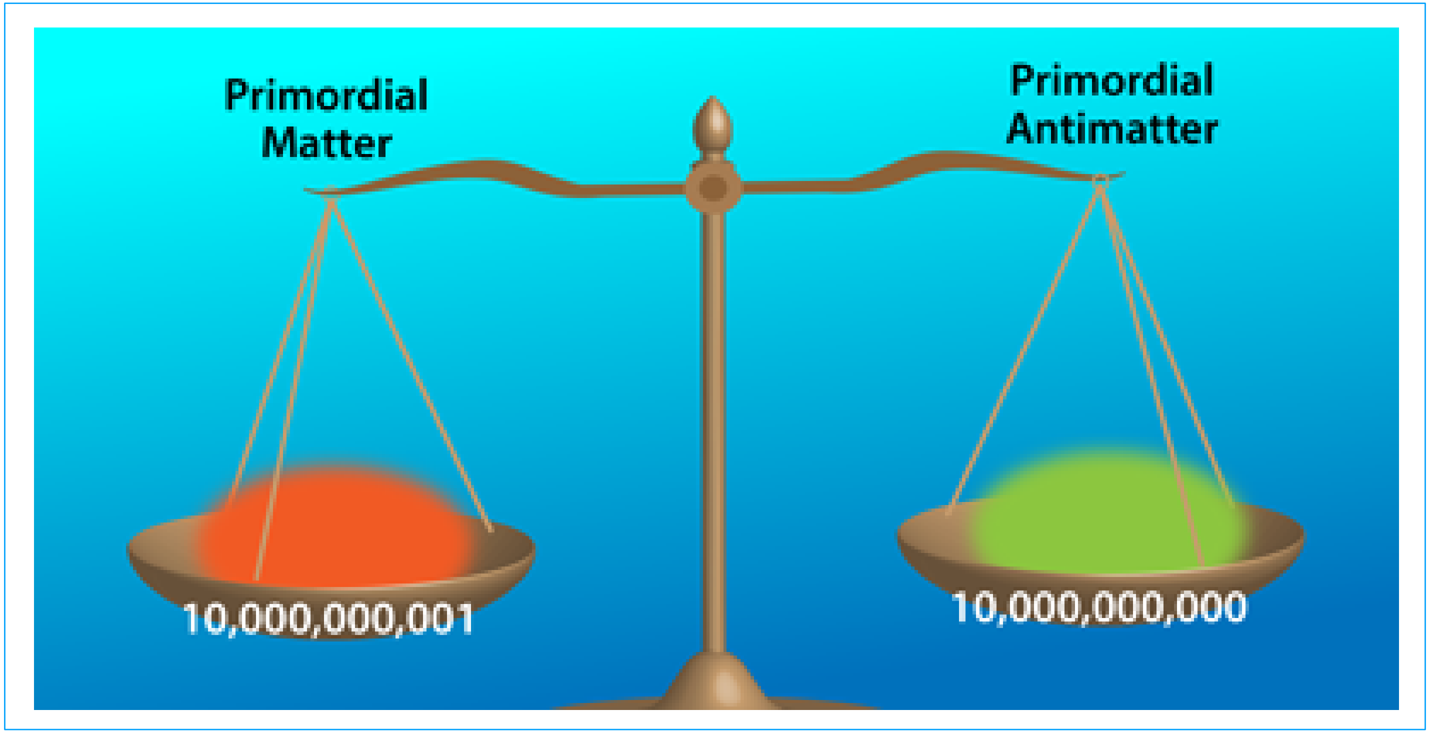
[1] W. Pauli, quoted repeatedly since 1931 [Dec 4, 1930], to be published shortly …, 1933 [(Gauthier-Villars, Paris, 1934)].

H. Bethe and R. Peierls, Nature 133, 532 (1934)

What did we learn since then?

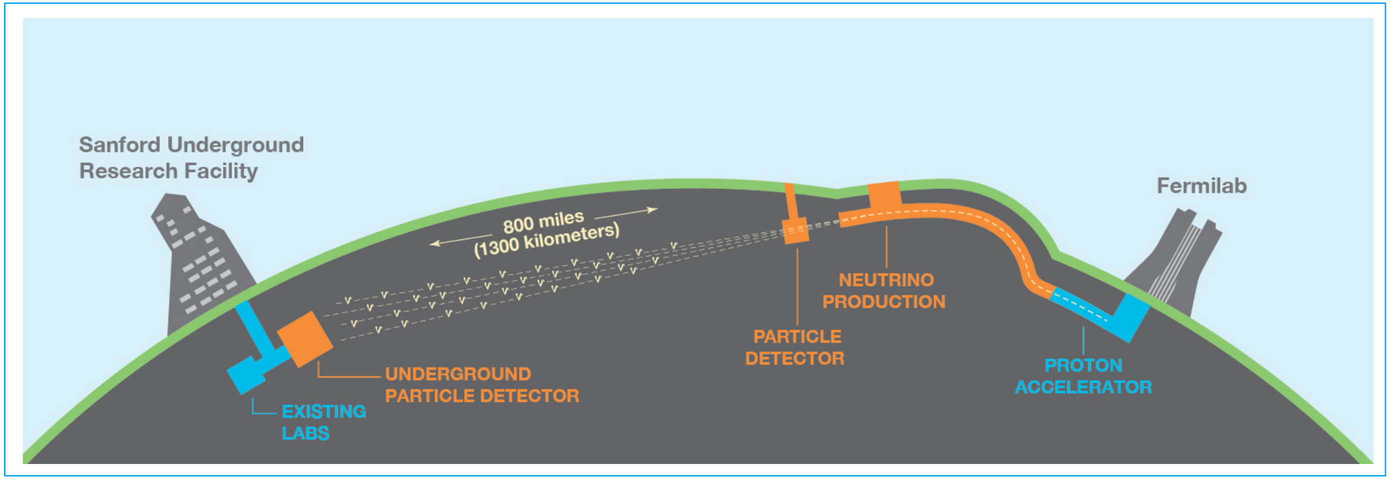
- First detection—Reines and Cowan (1956, 1995)
- More than one type of neutrino exists ($\nu_\mu \neq \nu_e$)—Lederman, Schwartz, and Steinberger (1962, 1988)
- Direct observation of ν_τ—DONUT Collab. (2001)
- Detection of supernova neutrinos—Kamiokande II, IMB, Baksan (1987, 2002)
- The solar model is correct—Davis (1970–1995, 2002) and SNO (2002, 2015)
- Neutrinos can oscillate—Super-Kamiokande (1998, 2015)
- Three mixing angles are nonvanishing—KamLAND (2002–2009, 2016), Daya Bay, RENO, Double Chooz (2012, 2016)

Matter-antimatter asymmetry



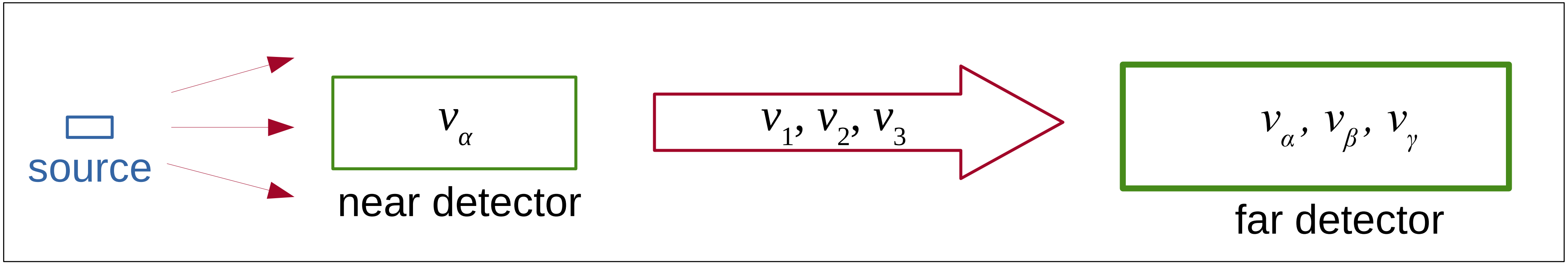
APS/Alan Stonebraker, https://physics.aps.org/articles/v8/s17

Deep Underground Neutrino Experiment (DUNE)



- Long-baseline physics—measurement of all oscillation parameters (including the mass hierarchy and δ_{CP}), test the 3-flavor framework
- Low-energy physics (supernova ν's, diffuse supernova ν's, solar ν's)
- Search for physics beyond the Standard Model

Neutrino oscillations in a nutshell



- ν's produced in a given flavor α (= e, μ, τ), mixture of mass eigenstates j (= 1, 2, 3)
- Different masses propagate with different phases, $e^{-iE_j t}$

$$tE_j = t\sqrt{\mathbf{p}^2 + m_j^2} = t|\mathbf{p}|\sqrt{1 + m_j^2/\mathbf{p}^2} \approx t|\mathbf{p}|\left(1 + \frac{m_j^2}{2\mathbf{p}^2}\right) \approx t|\mathbf{p}| + \frac{m_j^2 L}{2E_\nu}$$

- In the far detector, **mixture of mass eigenstates is different**—other flavors appear

Neutrino oscillations in a nutshell

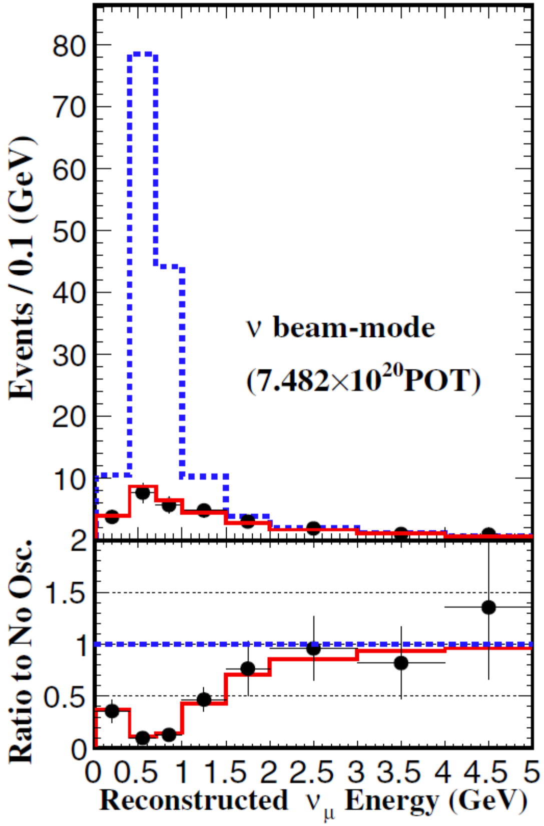
In the simplest case of two flavors

$$P(\nu_\alpha \to \nu_\alpha) = 1 - \sin^2 2\theta \sin^2 \left(\frac{\Delta m^2 L}{4E_\nu} \right)$$

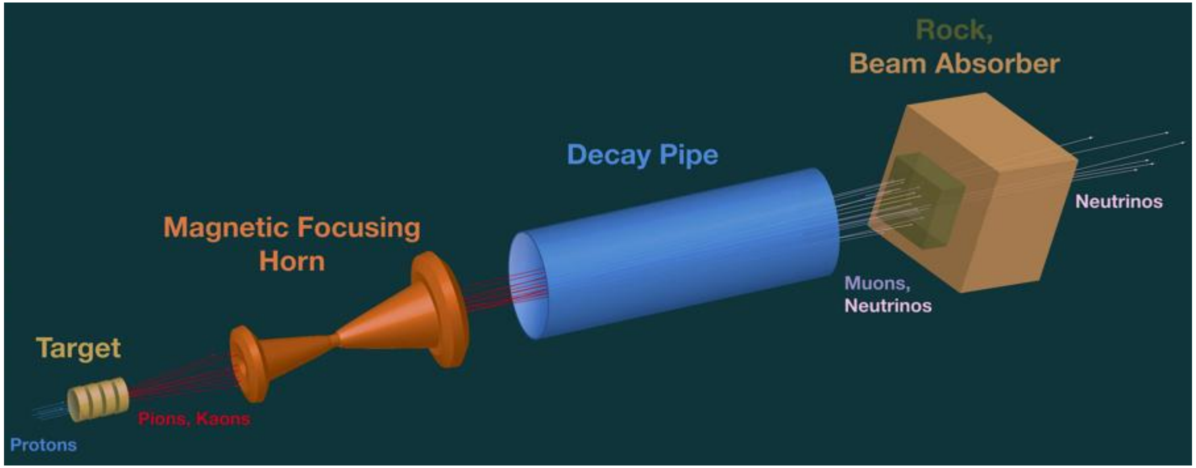
In the ratio of the observed to unoscillated events,

- the position of the dip determines Δm^2,
- its depth determines θ

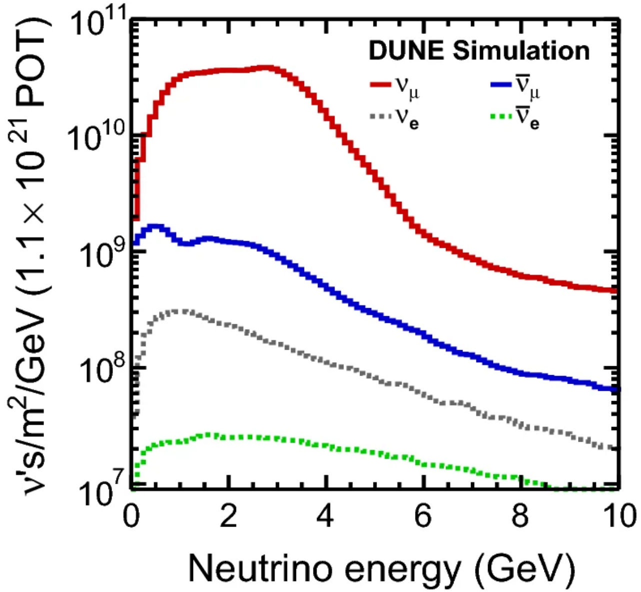
Abe *et al.* (T2K),
PRD 96, 011102(R) (2017)



How to make an accelerator neutrino beam



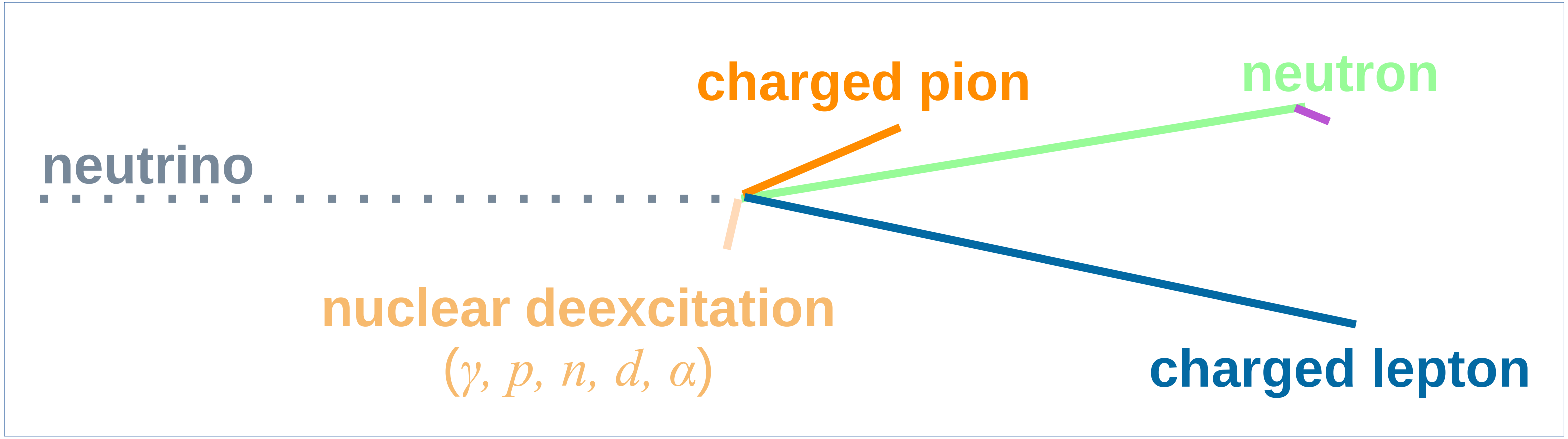
https://news.fnal.gov/2014/12/how-to-make-a-neutrino-beam/



Abi *et al.* (DUNE), EPJ C 978 (2020)

- Produce pions in proton collisions with the target.
- Focus them using horn(s).
- Let pions decay, $\pi \rightarrow \mu + \nu_\mu$, and absorb muons before they decay, $\mu \rightarrow \nu_\mu + e + \overline{\nu_e}$.

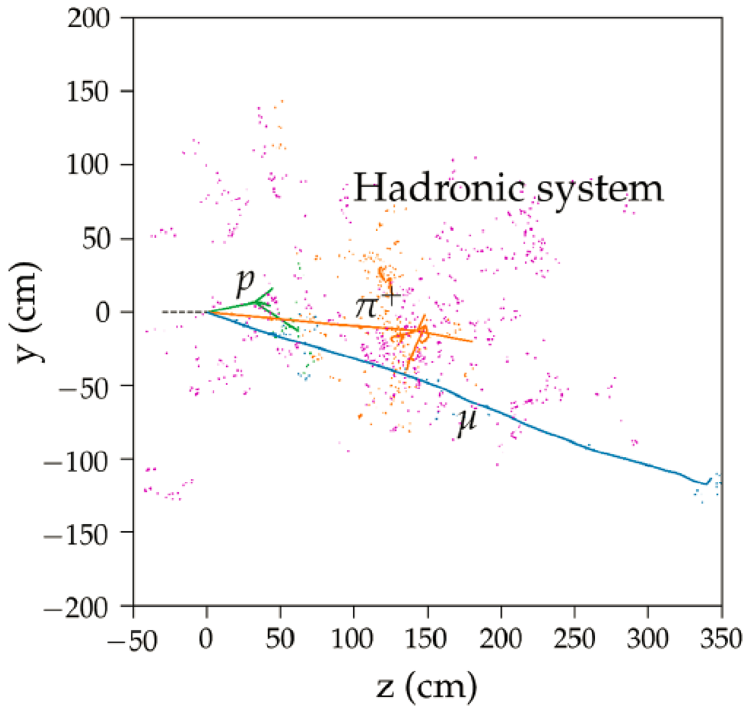
Neutrino energy reconstruction



Neutrino energy is converted to

- the kinetic energies of the knocked-out nucleons,
- the total energies of leptons, pions, and gammas,
- nuclear breakup

GENIE+FLUKA simulation of a 4-GeV ν_μAr event

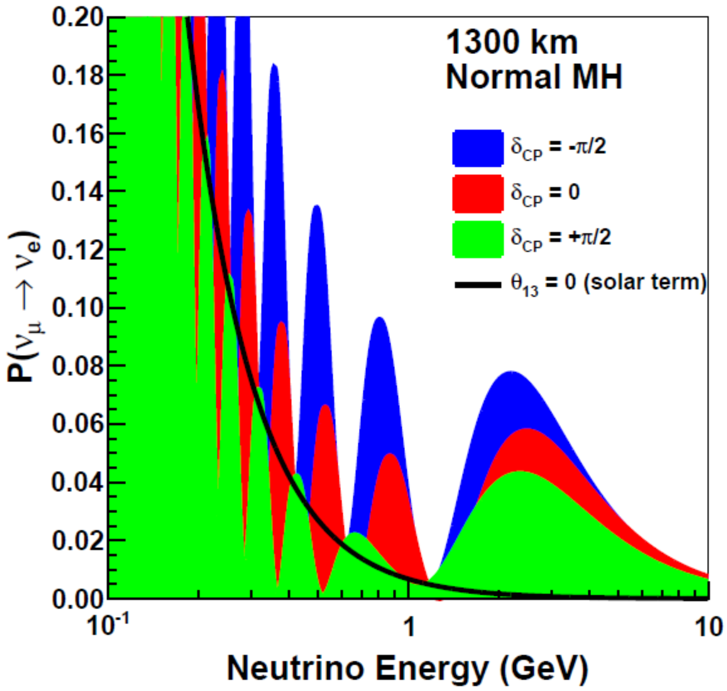


A. Friedland & S.W. Li, PRD 99, 036009 (2019)

Multiply differential cross sections required for energy reconstruction.

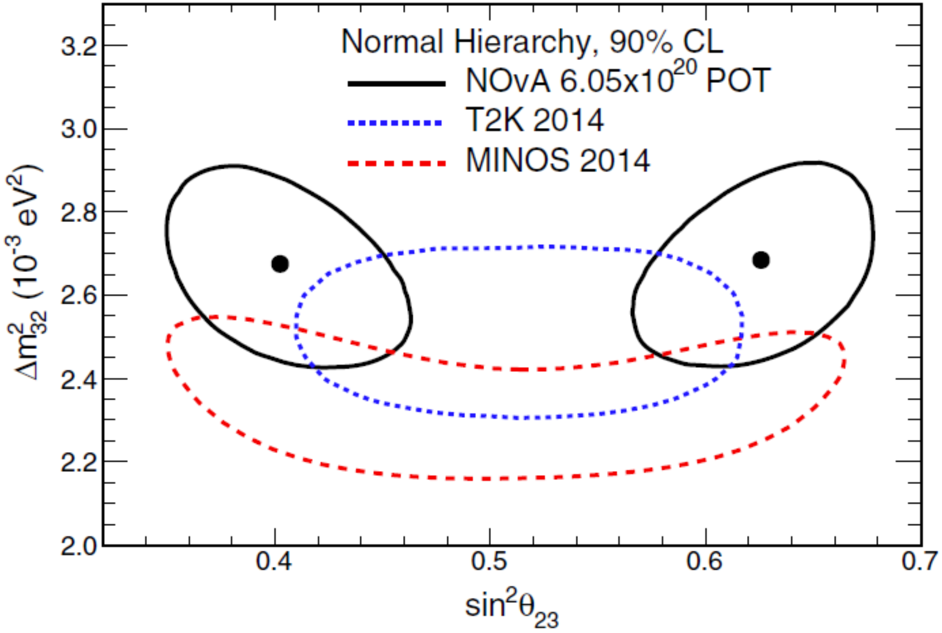
MC Generators in long-baseline neutrino physics

- Main goal: extract the ν & $\bar{\nu}$ oscillation probabilities.
- Polychromatic beams, neutrino energy reconstructed from visible energy deposited by interaction products.
- Calorimetric reconstruction of neutrino energy.
- Sizable contributions of hadrons. Neutrons' energy estimate heavily dependent on Monte Carlo.
- Accuracy of simulations translates into the accuracy of the extracted oscillation parameters.
- We are no longer after $\mathcal{O}$ (1) effects, **without reliable cross sections precise measurements cannot succeed.**

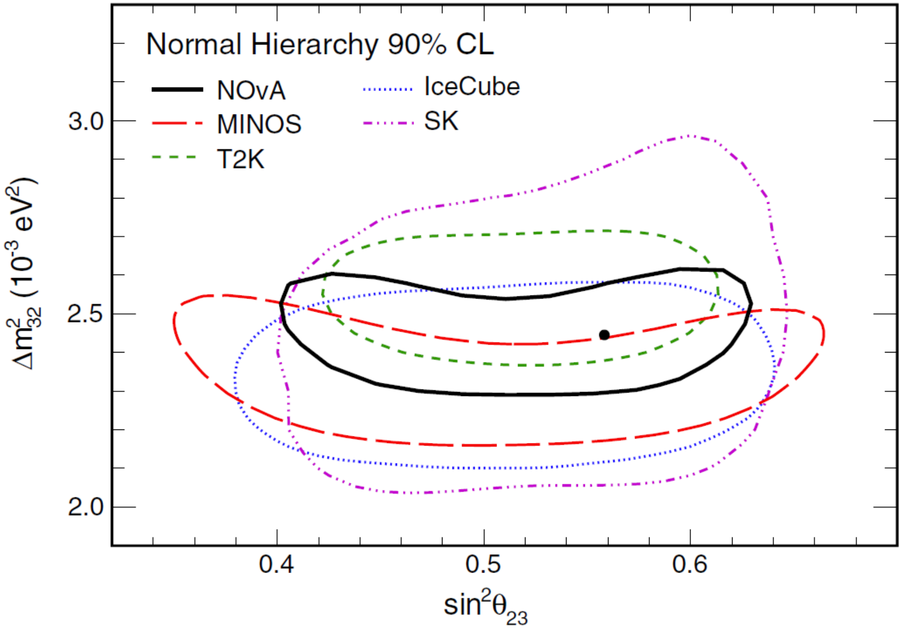


DUNE Collaboration, arXiv:1512.06148

Concrete example: NOvA



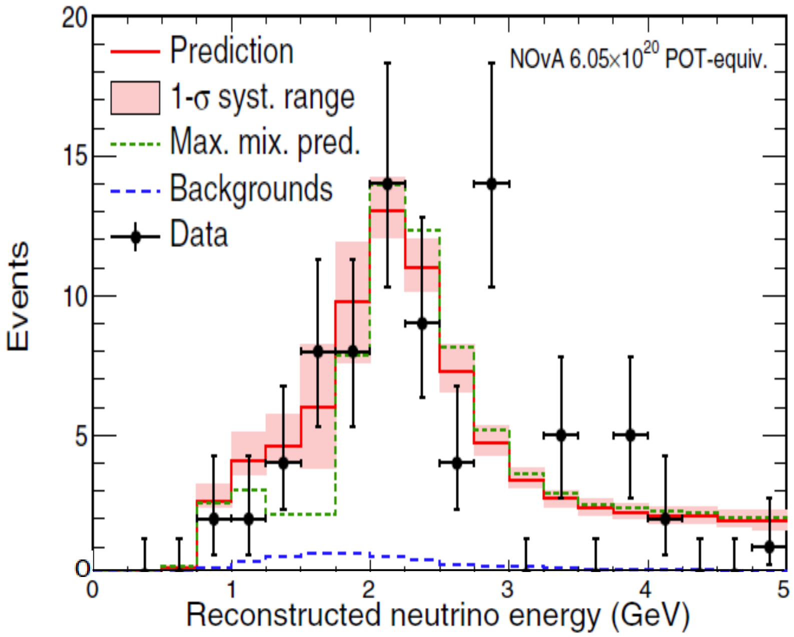
Acero *et al.* (NOvA), PRL 118, 151802 (2017)



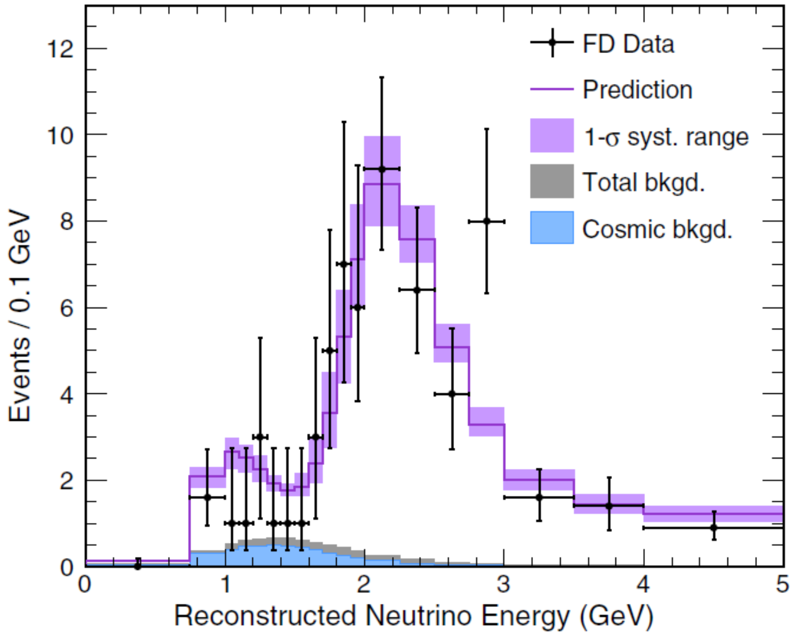
Acero *et al.* (NOvA), PRD 98, 032012 (2018)

"This change was caused by three changes ... The largest effect was due to new simulations and calibrations."

Concrete example: NOvA



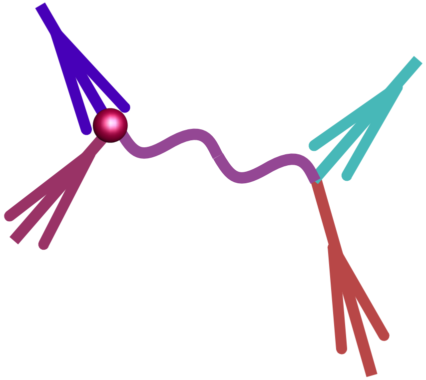
Acero *et al.* (NOvA), PRL 118, 151802 (2017)



Acero *et al.* (NOvA), PRD 98, 032012 (2018)

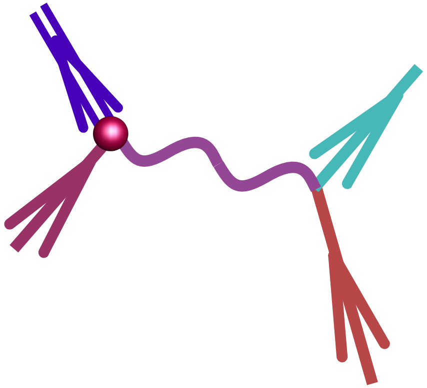
"This change was caused by three changes ... The largest effect was due to new simulations and calibrations."

Neutrino scattering at GeV energies



quasielastic scattering

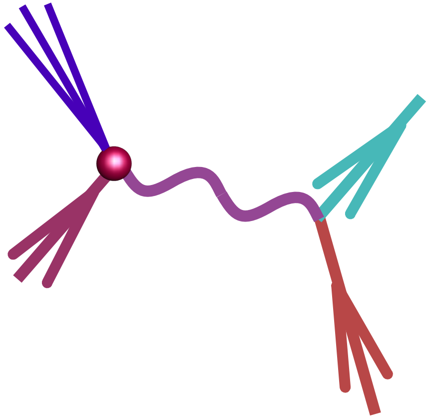
$$\nu_l + n \rightarrow l^- + p$$
$$\bar{\nu}_l + p \rightarrow l^+ + n$$



resonance production

$$\nu_l + n \rightarrow l^- + \Delta^+$$
$$\nu_l + p \rightarrow l^- + \Delta^{++}$$
$$\bar{\nu}_l + n \rightarrow l^+ + \Delta^-$$
$$\bar{\nu}_l + p \rightarrow l^+ + \Delta^0$$

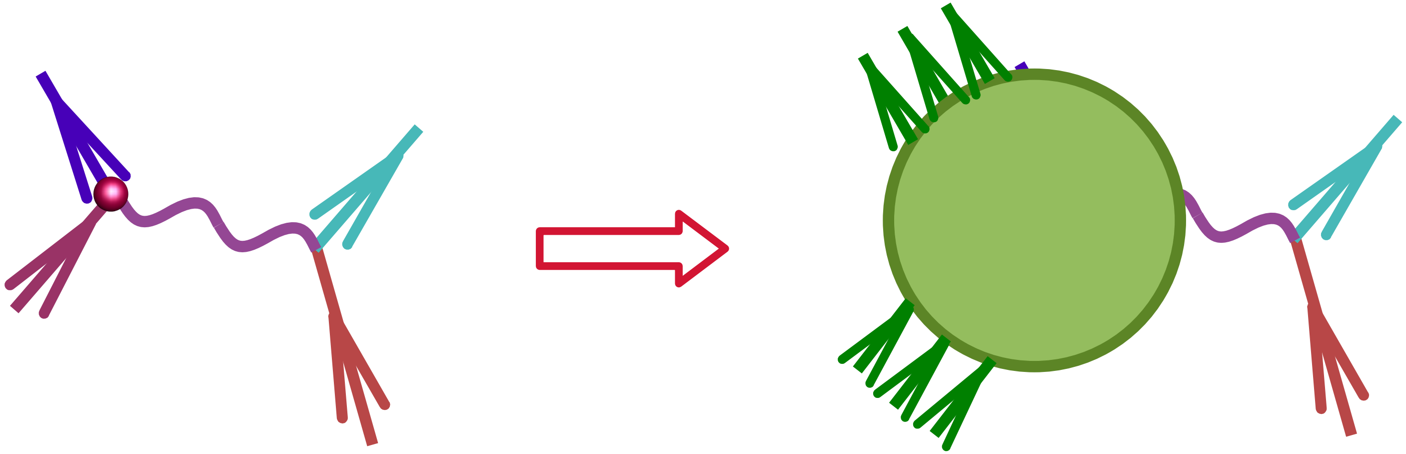
...



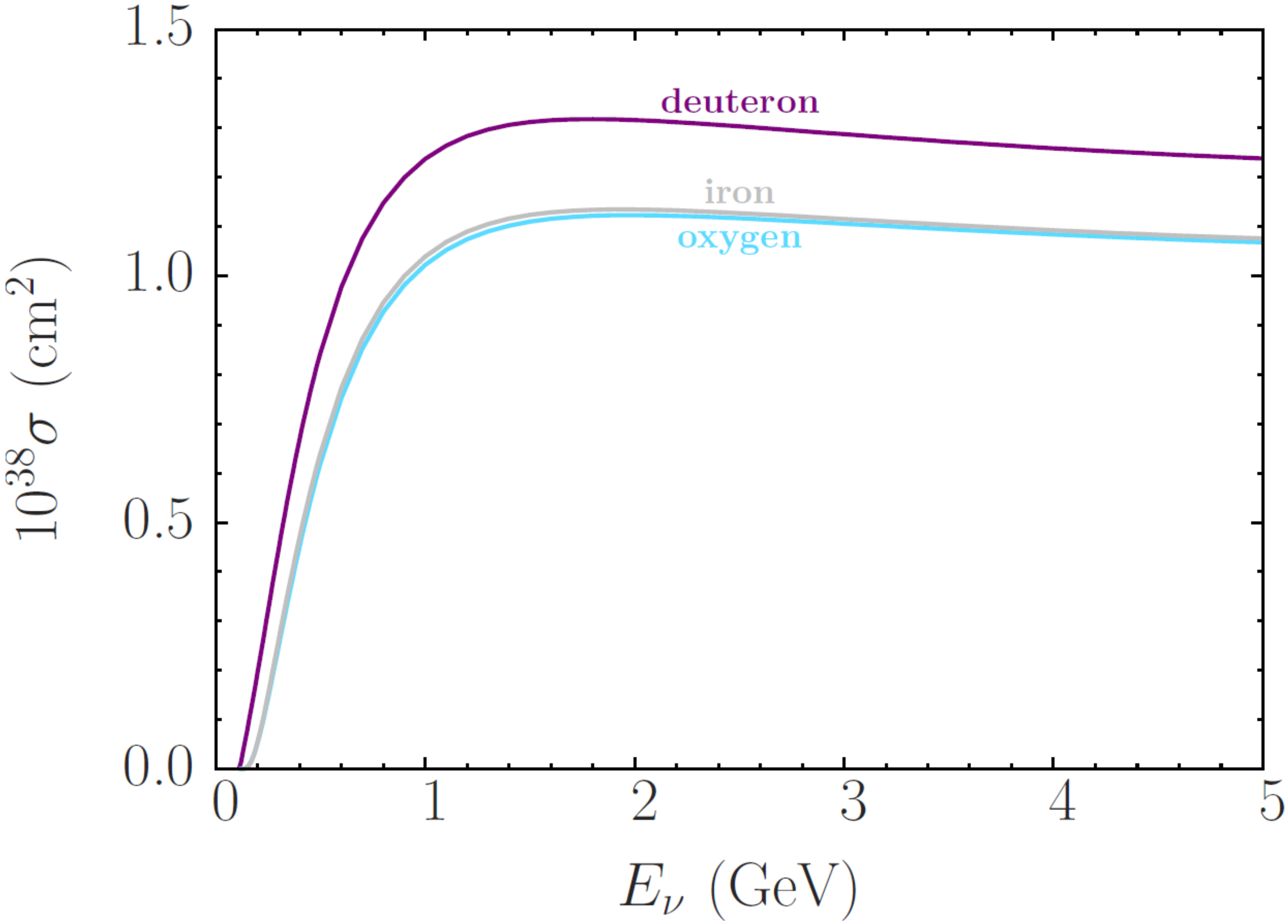
deep-inelastic scattering

$$\nu_l + N \rightarrow l^- + N' + n\pi$$
$$\bar{\nu}_l + N \rightarrow l^- + N' + n\pi$$

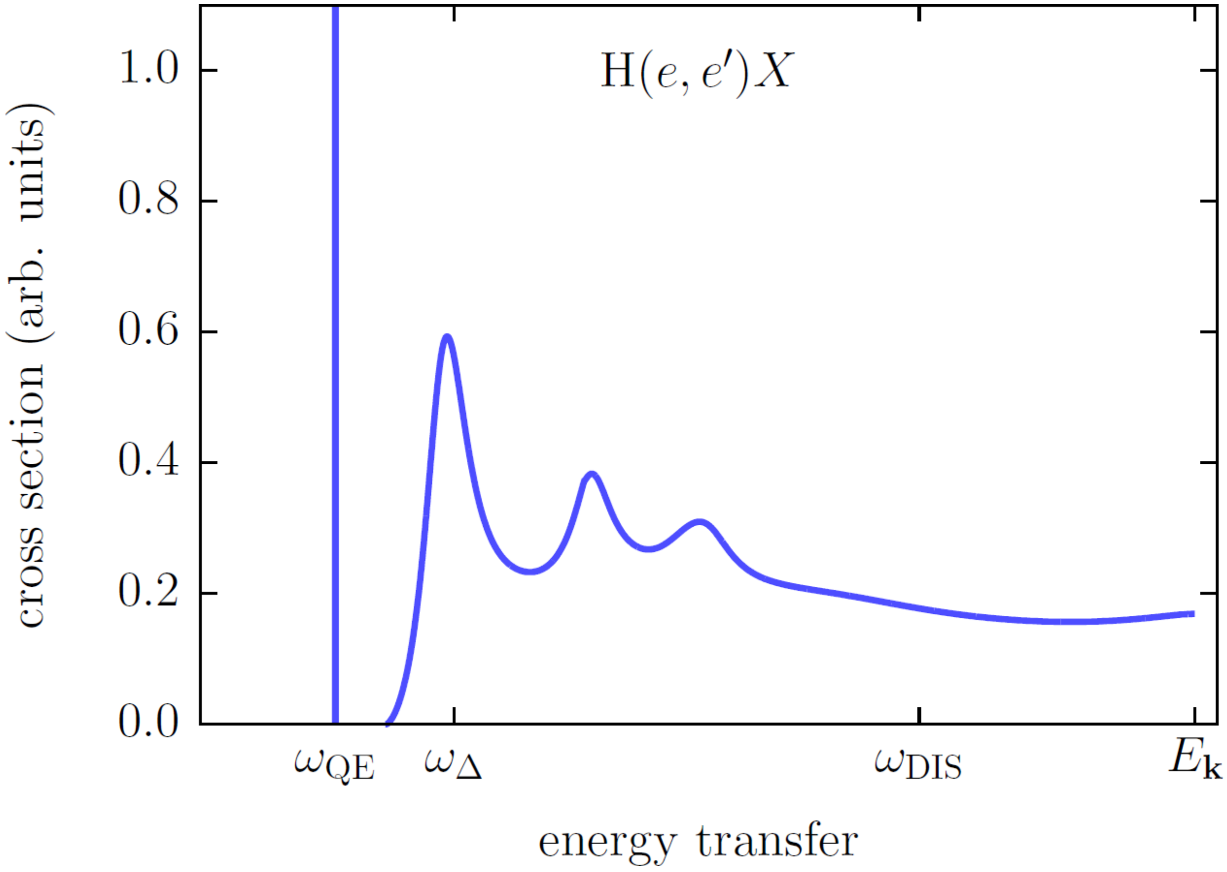
Nuclear effects



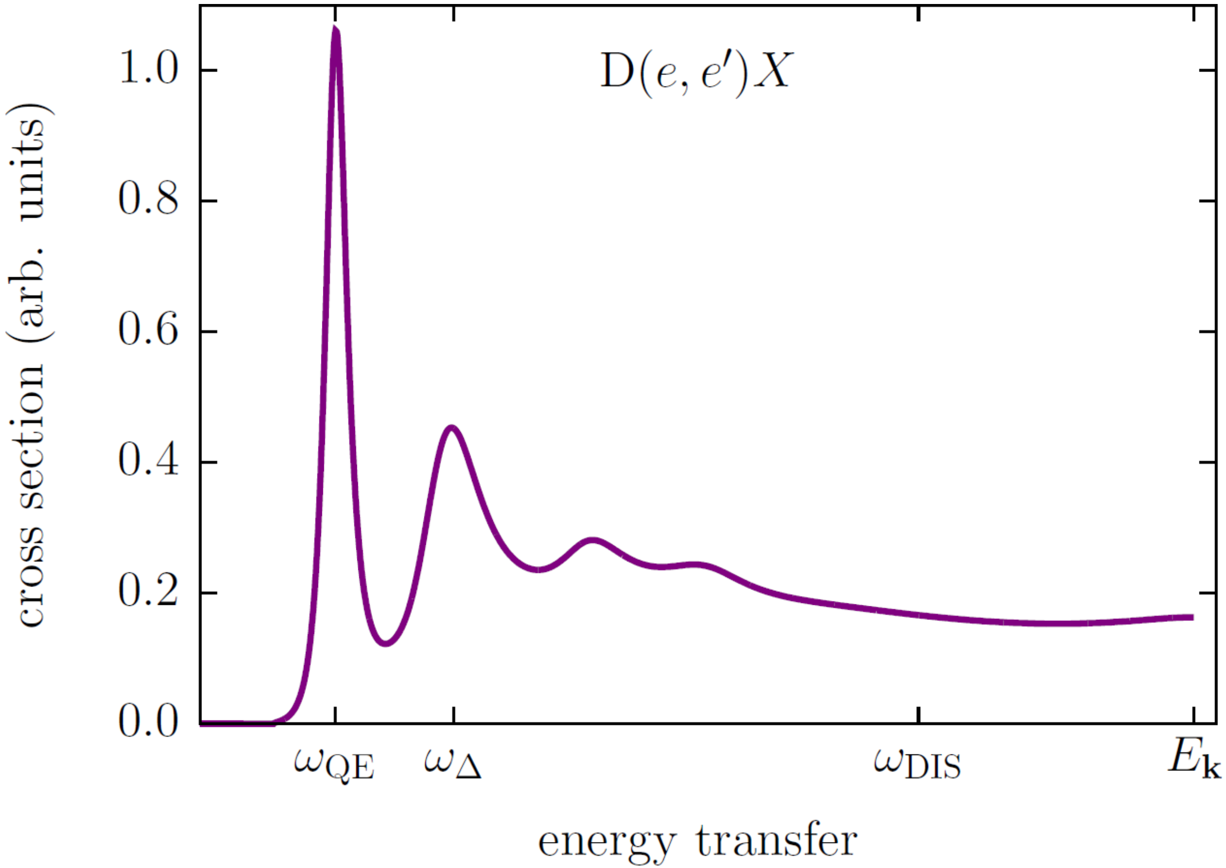
Total (QE) cross section



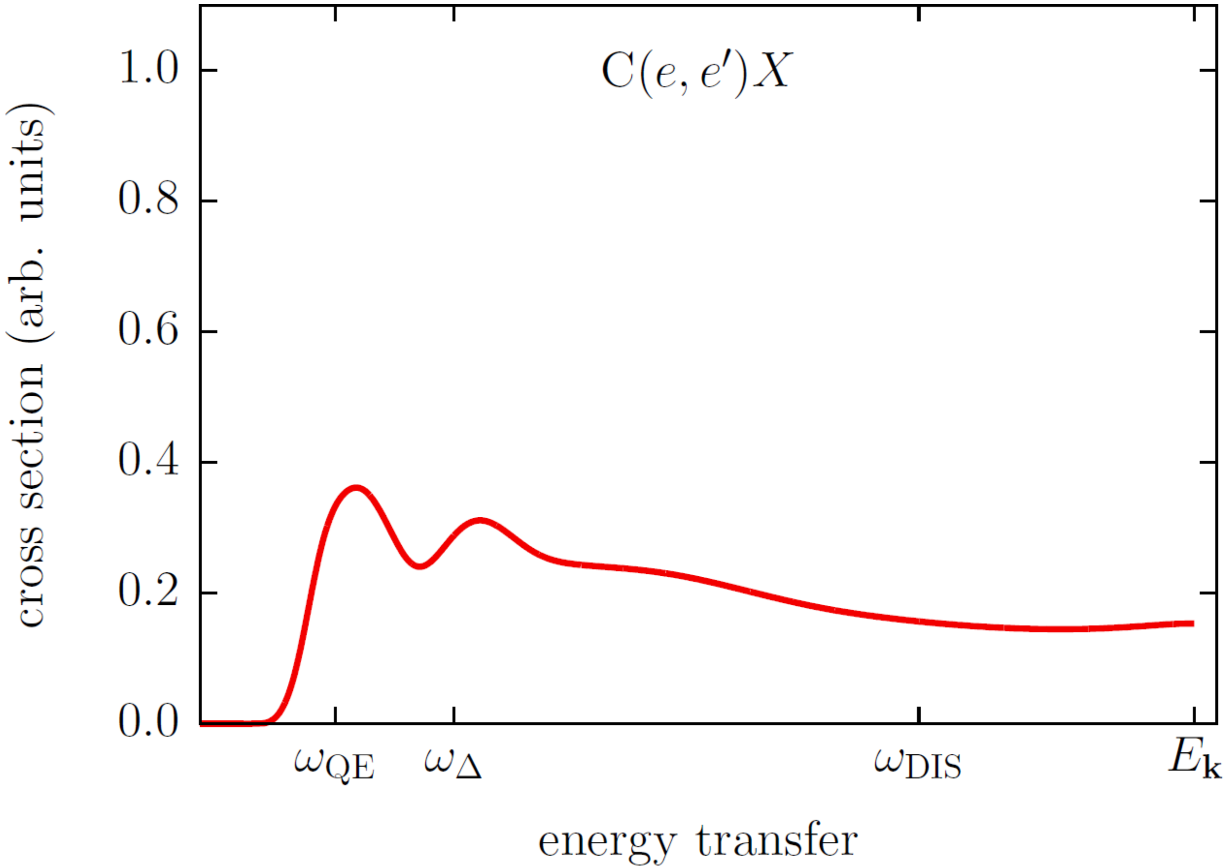
Double differential cross section



Double differential cross section



Double differential cross section



Current situation

"… **nuclear models available to modern neutrino experiments give similar results** … **none of which is confirmed by the data.** … More theoretical work is needed to correctly model nuclear effects … from the quasielastic to the deep inelastic regime."

B. G. Tice *et al.* (MINERvA), PRL 112, 231801 (2014)

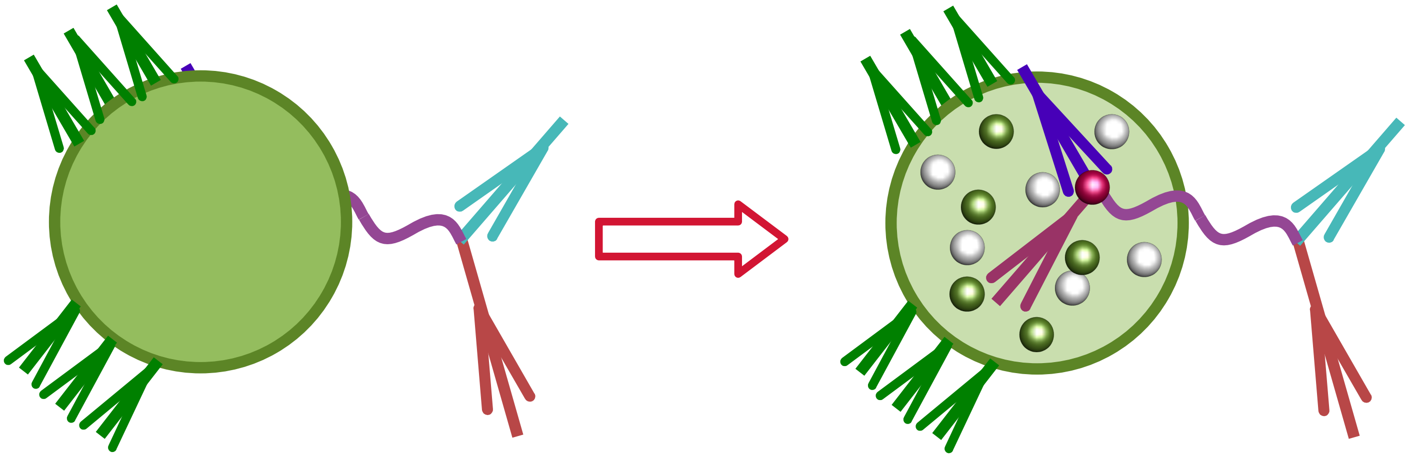
"The double- and single-differential cross sections show similar tensions with the model predictions. These results demonstrate that **improvements will need to be made to neutrino-interaction models if precision neutrino oscillation experiments hope to better constrain the systematics** …"

A. Filkins *et al.* (MINERvA), PRD 101, 112007 (2020)

Impulse approximation

At relevant kinematics, the dominant process of neutrino-nucleus interaction is **scattering off a single nucleon**, with the remaining nucleons acting as a spectator system.

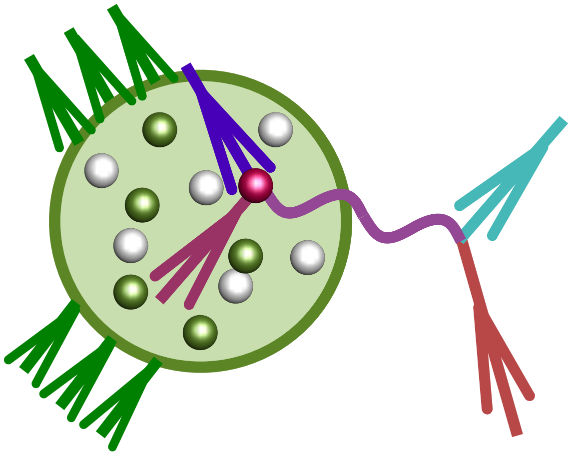
This description is valid when the momentum transfer |**q**| is high enough (|**q**| $\gtrsim$ 200 MeV).



Impulse approximation

To calculate the neutrino-argon cross sections we need to know

- elementary cross sections (QE, resonant pion production, DIS ...)
- proton and neutron spectral functions (distributions of the initial momenta and energies, correlations between nucleons, …)
- final-state interactions (nuclear transparency, optical potentials)
- hadronization



Electrons and neutrinos

For scattering in a given angle and energy, ν's and e's differ almost exclusively due to the elementary cross sections.

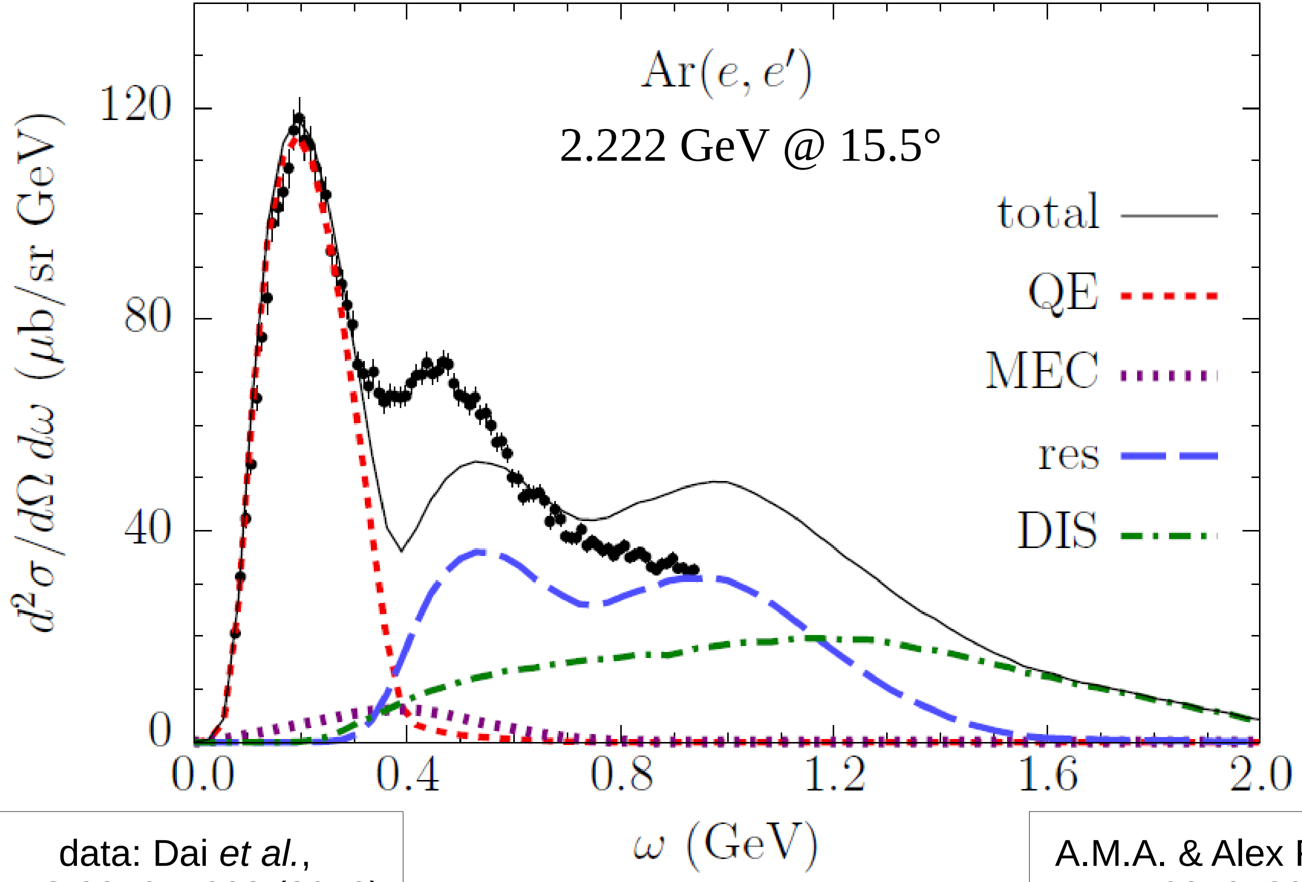
Electron-scattering data can provide information on

- the vector contributions to elementary neutrino cross sections
- proton and neutron spectral functions (Ar & Ti targets)
- hadronization (H & D targets)
- final-state interactions (Ar & Ti + H & D targets)

Electron data allow MC validation, reduction of systematic uncertainties, as well as their rigorous determination.

A.M.A., A. Friedland, S. W. Li, O. Moreno, P. Schuster, N. Toro & N. Tran, PRD 101, 053004 (2020)

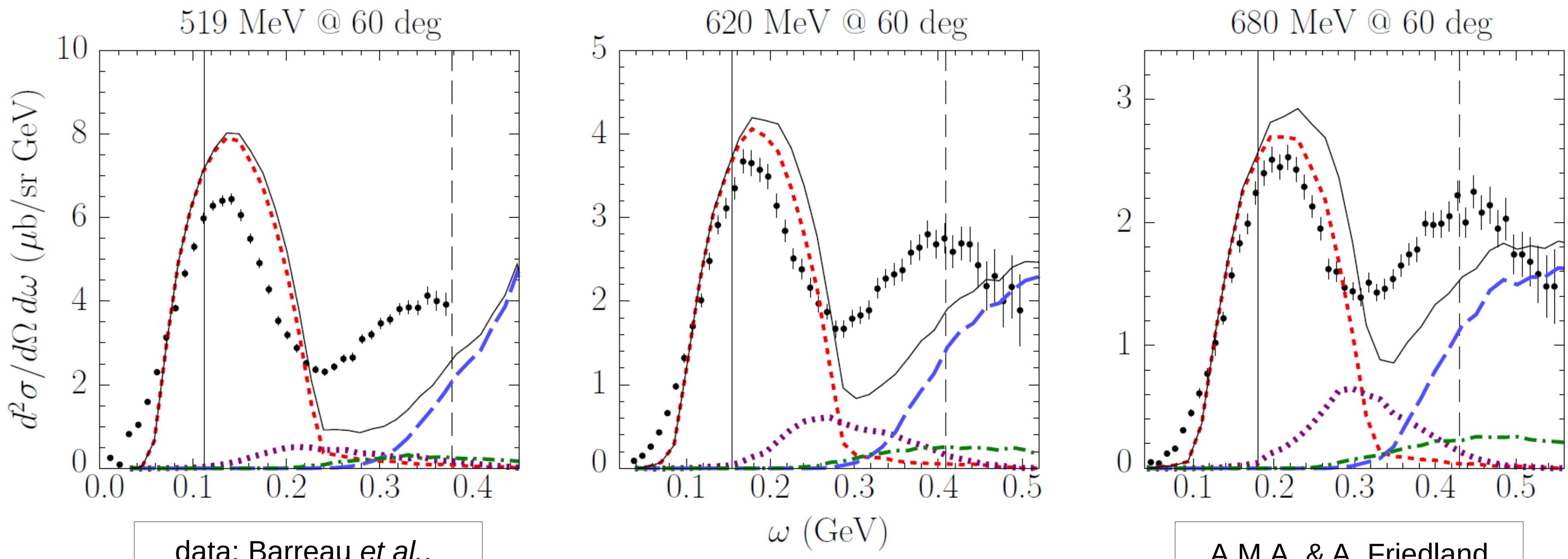
Current situation



data: Dai *et al.*, PRC 99, 054608 (2019)

A.M.A. & Alex Friedland, PRD 102, 053001 (2020)

Current situation



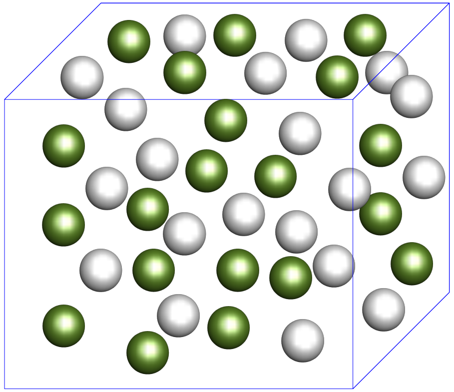
data: Barreau *et al.*,
NPA 402, 515 (1983)

A.M.A. & A. Friedland,
PRD 102, 053001 (2020)

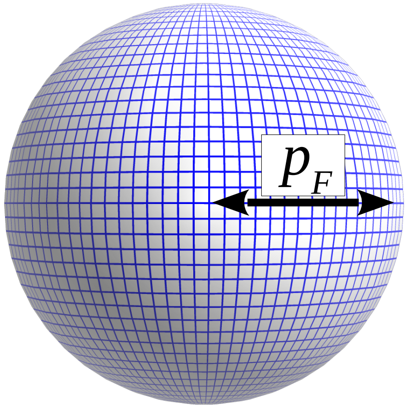
Fermi gas

Nucleus treated as a fragment of non-interacting infinite nuclear matter of constant density.

Eigenstates have definite momenta and energies $E_p = \sqrt{M^2 + \boldsymbol{p}^2} - \epsilon$.

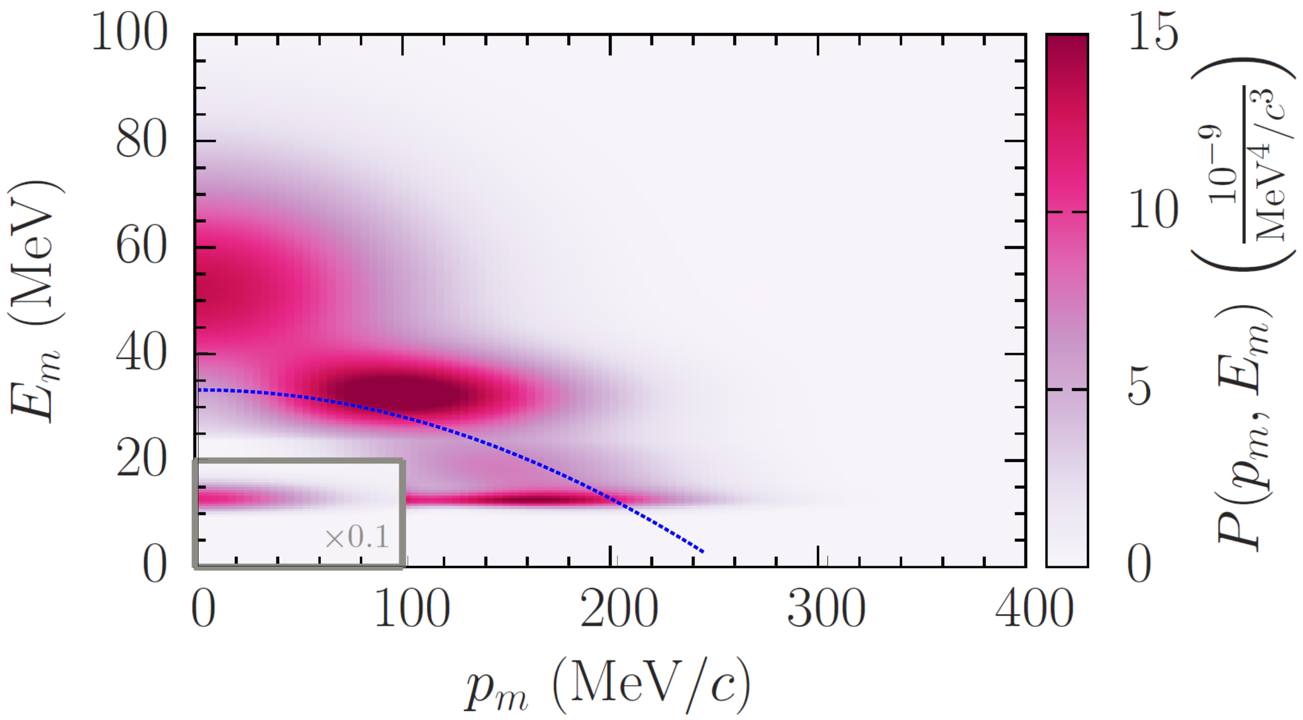


Coordinate space

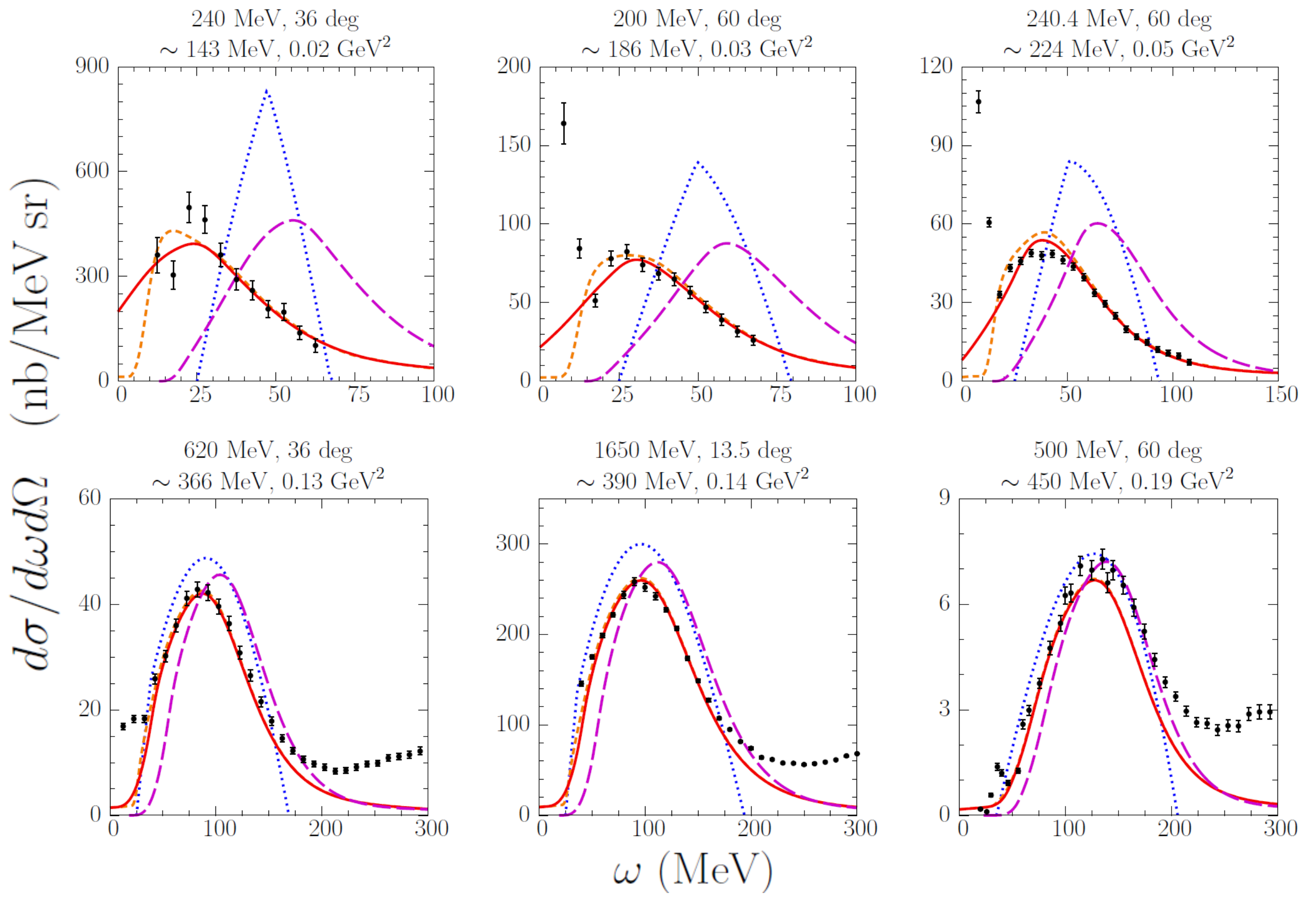


Momentum space

Fermi gas vs. spectral function

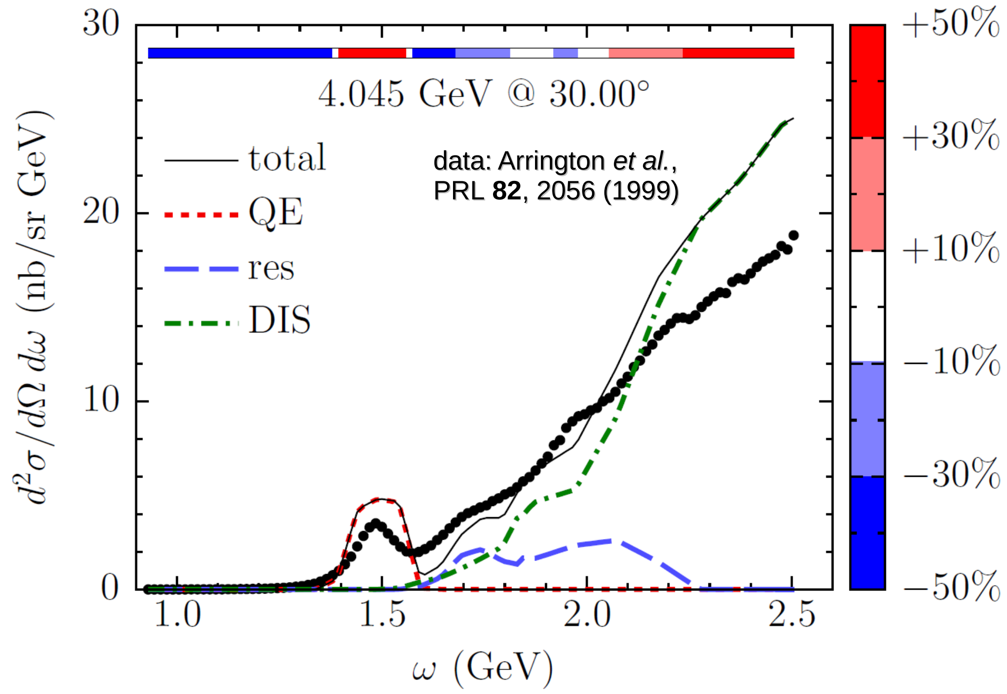


Realistic description of the nucleus: C(*e*,*e*')

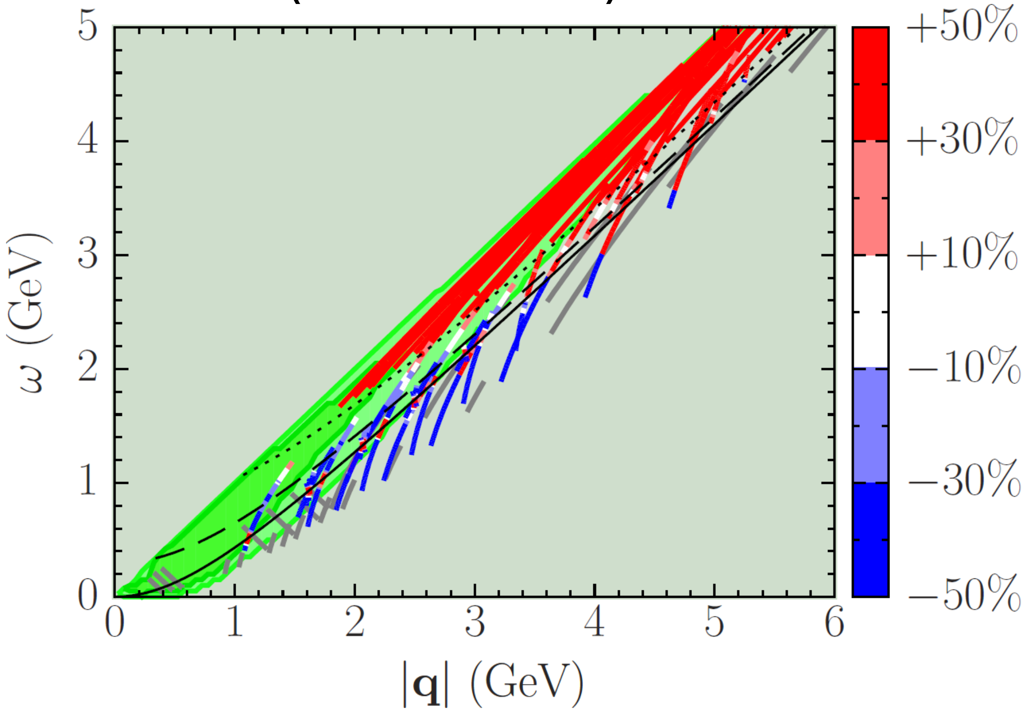


A.M.A., O. Benhar & M. Sakuda, PRD 91, 033005 (2015)

D(*e*,*e’*) in the Monte Carlo generator GENIE

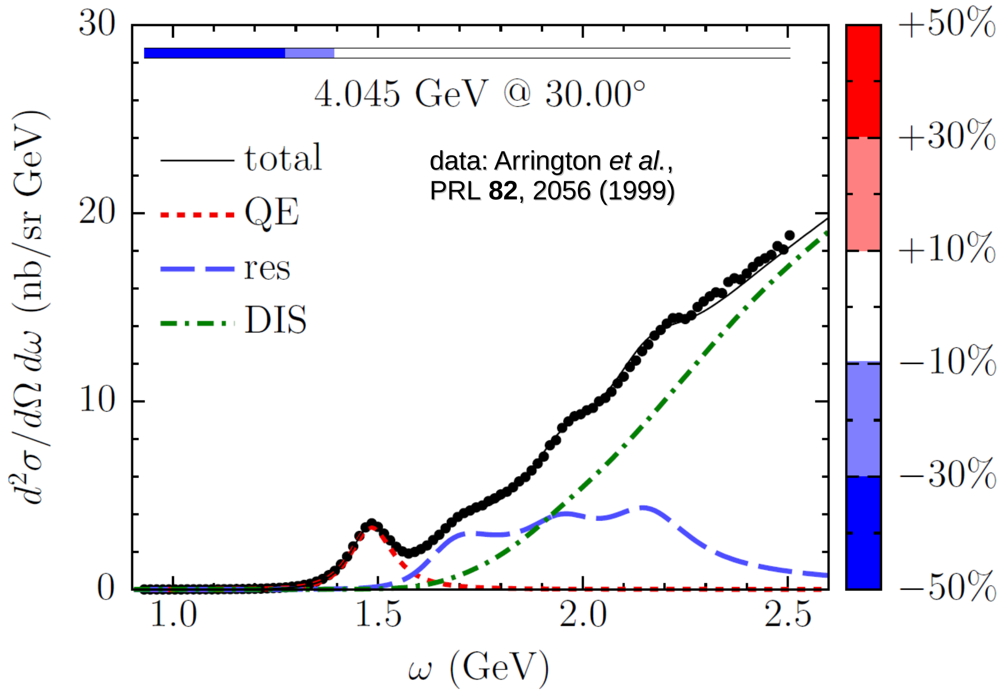


(GENIE − data)/data

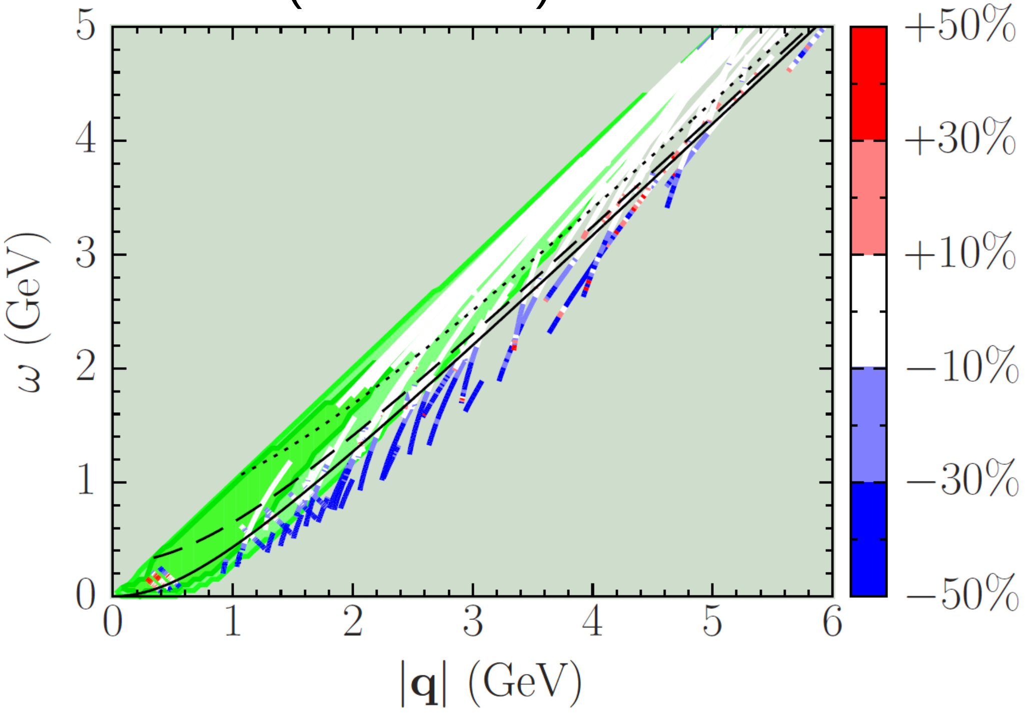


Adopted from
A.M.A. & Alex Friedland, PRD 102, 053001 (2020)

Realistic description of D(*e*,*e’*)

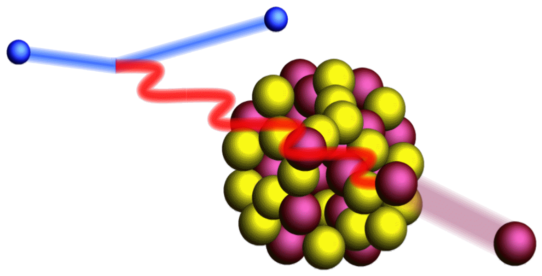


(calc − data)/data



E12-14-012 in JLab: (*e*,*e'*) and (*e*,*e'p*) on Ar and Ti

Aim: Obtaining the experimental input indispensable to construct the argon spectral function, thus paving the way for a reliable estimate of the neutrino cross sections in DUNE. In addition, stimulating a number of theoretical developments, such as the description of final-state interactions.
[Benhar *et al.*, arXiv:1406.4080]

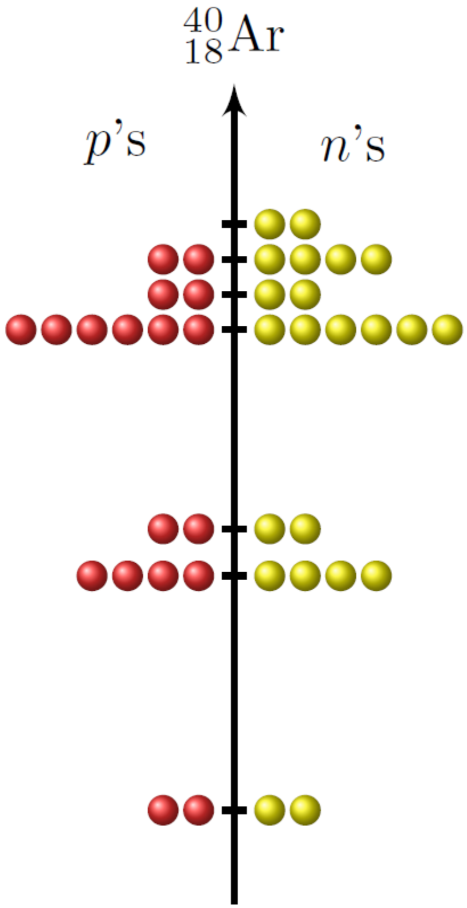


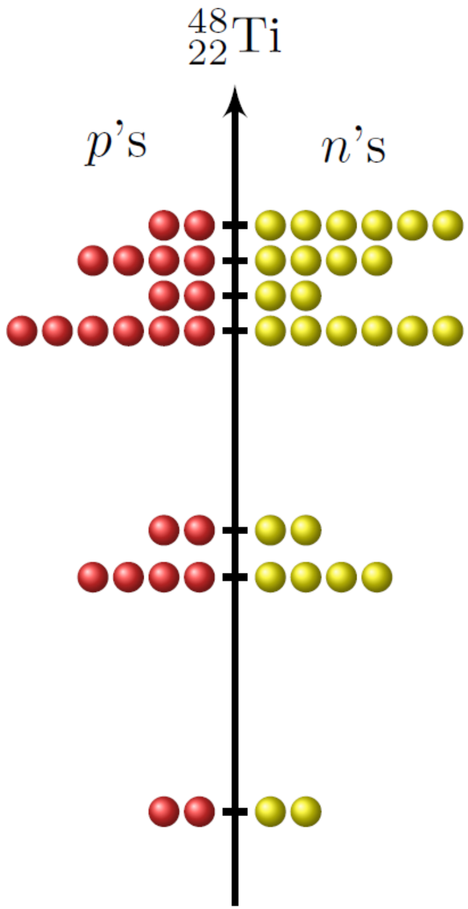
$E_e = 2.222$ GeV

	E'_e (GeV)	θ_e (deg)	$\|\mathbf{p}'\|$ (MeV)	$\theta_{p'}$ (deg)	$\|\mathbf{q}\|$ (MeV)	p_m (MeV)	E_m (MeV)
kin1	1.777	21.5	915	−50.0	865	50	73
kin2	1.716	20.0	1030	−44.0	846	184	50
kin3	1.799	17.5	915	−47.0	741	174	50
kin4	1.799	15.5	915	−44.5	685	230	50
kin5	1.716	15.5	1030	−39.0	730	300	50

First, exploratory analyses of the full datasets

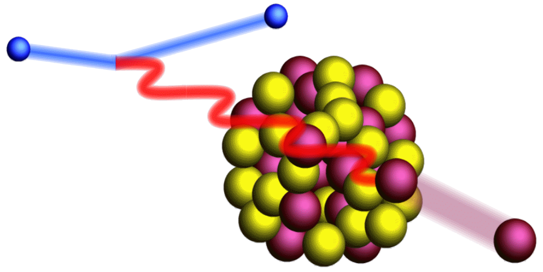
Why titanium?



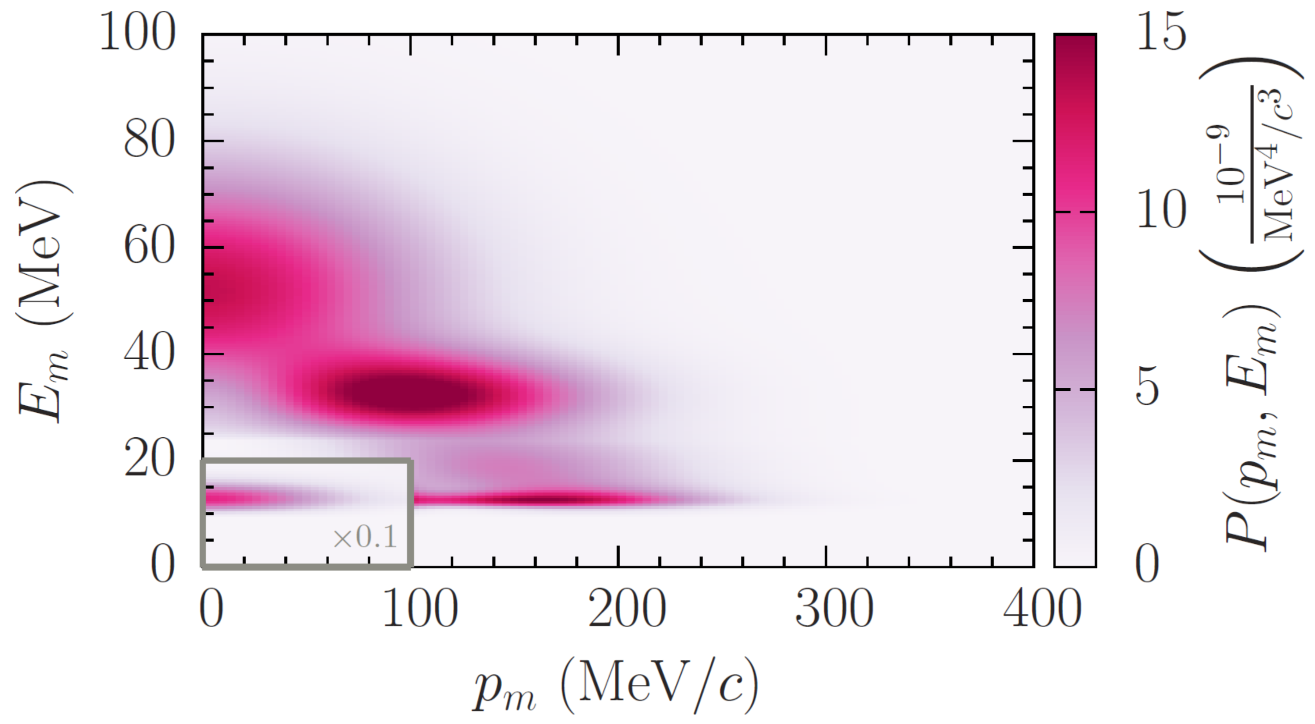


This analysis: extraction of the spectral function

The proton spectral function $P(p_m, E_m)$ describes the probability distribution of removing a proton of momentum p_m from the target nucleus, leaving the residual system with excitation energy $E_m - E_{thr}$, with E_{thr} being the proton emission threshold.

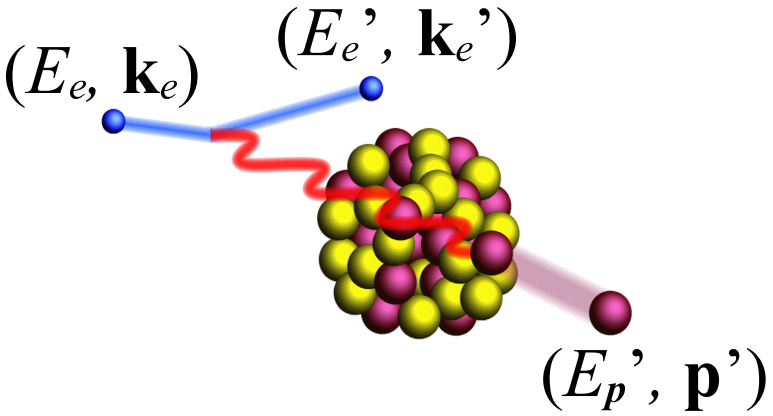


This analysis: extraction of the spectral function



Universal property of the nucleus, independent of the interaction.

Missing energy E_m and missing momentum $\mathbf{p}_m$



$$E_e + M_A = E_e' + E_p' + E^*_{A-1}$$

known

determined

$$\mathbf{k}_e + 0 = \mathbf{k}_e' + \mathbf{p}' + \mathbf{p}_{A-1}$$

In general,

$$E^*_{A-1} = \sqrt{(M_A - M + E_m)^2 + p^2_{A-1}}$$

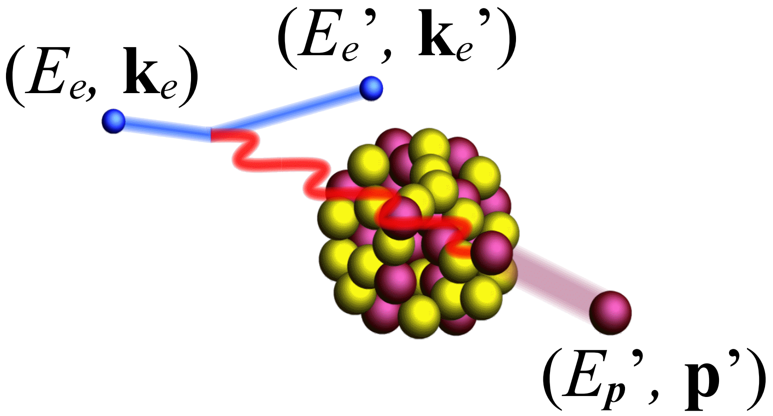
$E_m - E_{\mathrm{thr}}$ is the excitation energy of ^{39}Cl

Without final state interactions

$$-\,\mathbf{p}_{A-1} = \mathbf{p}_m$$

is the initial proton momentum

Missing energy E_m and missing momentum $\mathbf{p}_m$



$$E_e + M - \underline{E_m} = E_{e'} + E_p$$

known missing

$$\mathbf{k}_e + \underline{\mathbf{p}_m} = \mathbf{k}_{e'} + \mathbf{p}'$$

For negligible recoil energy,

$$E^*_{A-1} = M_A - M + E_m$$

$E_m - E_{\mathrm{thr}}$ is the excitation energy of ^{39}Cl

Without final state interactions

$$-\mathbf{p}_{A-1} = \mathbf{p}_m$$

is the initial proton momentum

Spectral function for complex nuclei

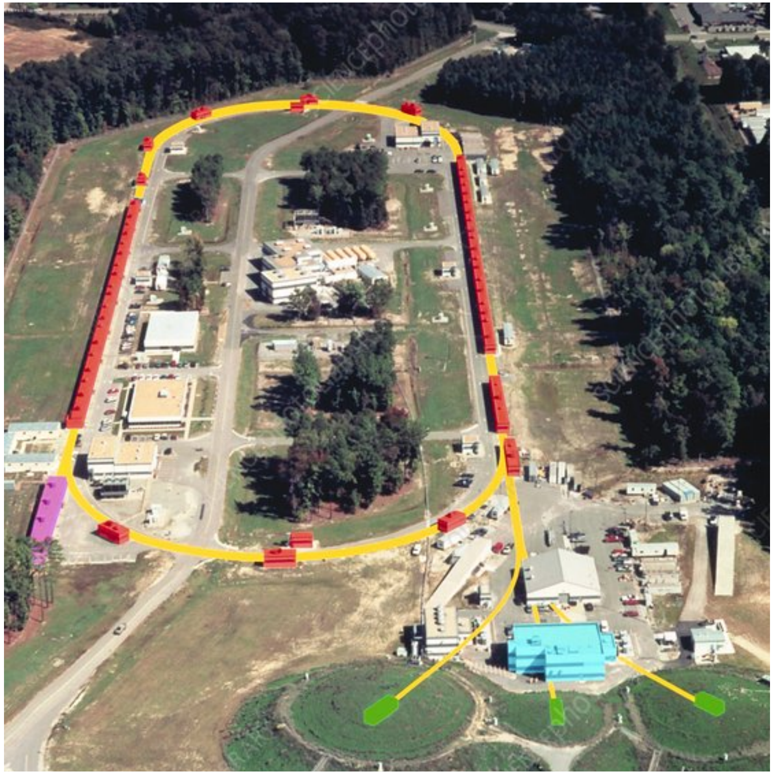
Mean-field part

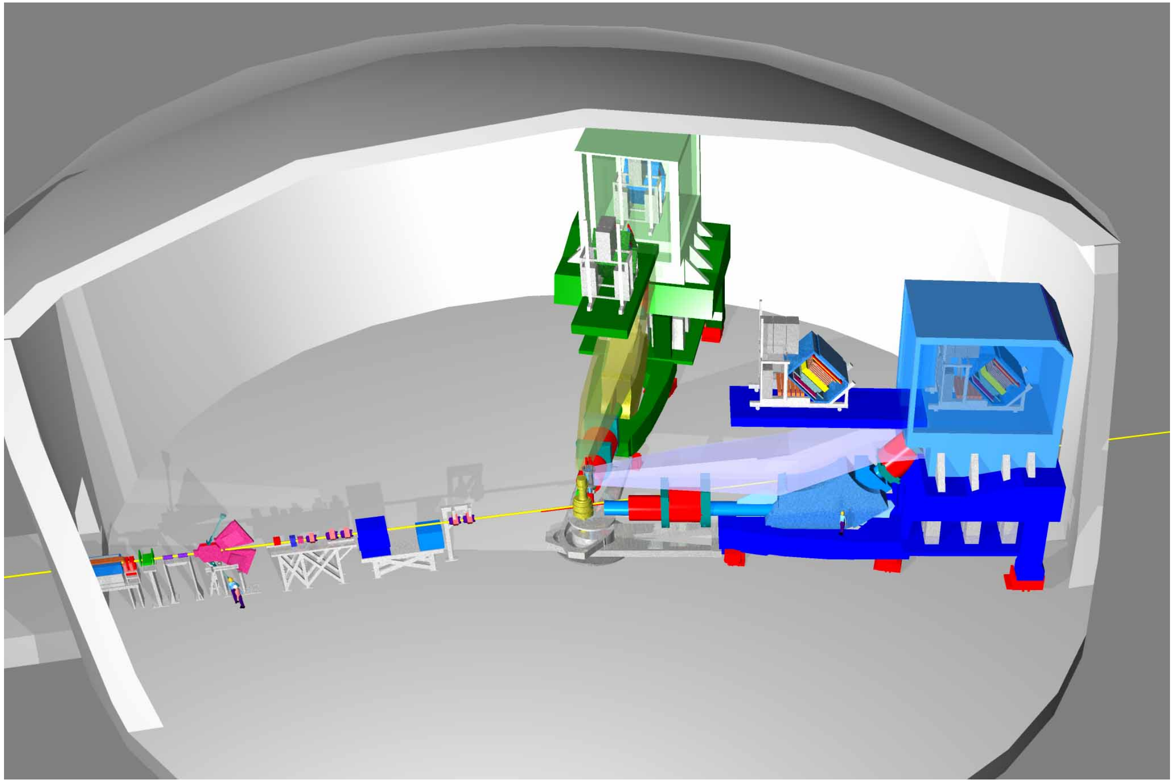
- describes the shell structure
- can be determined from experimental data
- 70–80% of nucleons

Correlated part

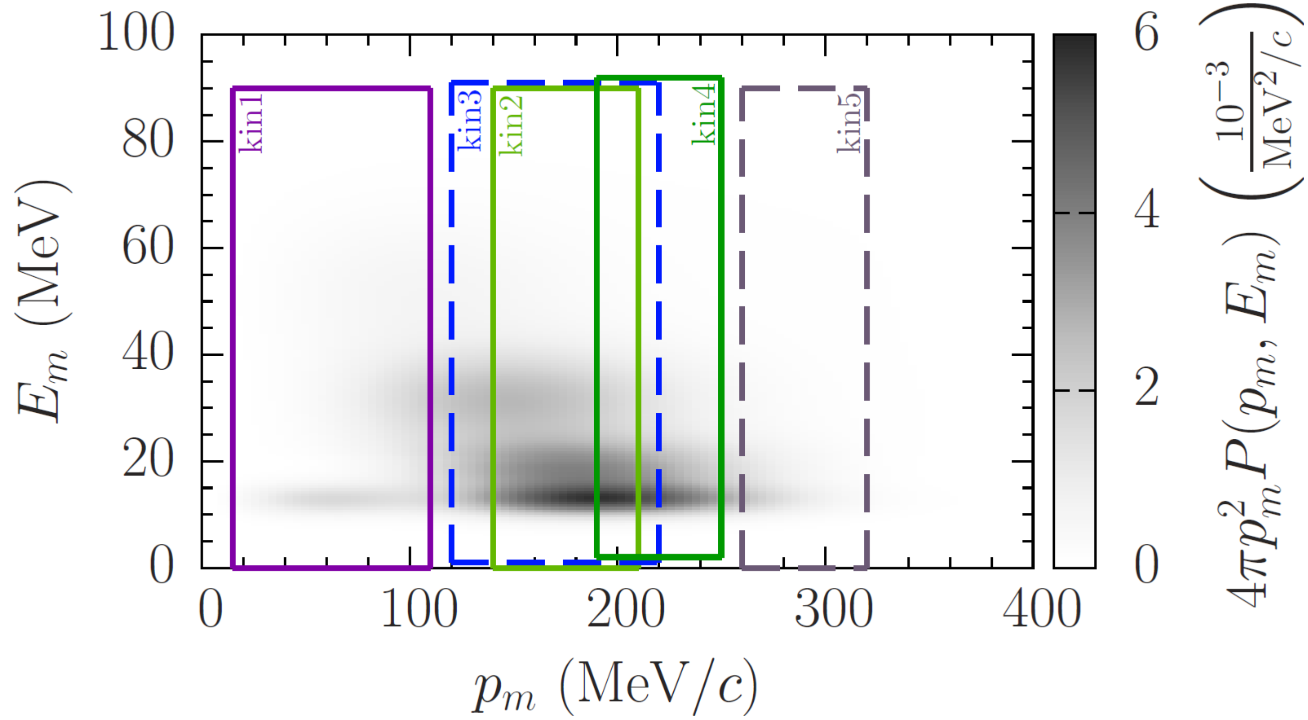
- describes correlated nucleons
- easier to determine from theoretical estimates

Jefferson Laboratory Hall A





^{40}Ar(e,$e'p$) in E12-14-012



(*e*,*e'p*) cross section

elementary cross section

nuclear transparency

$$\frac{d^4\sigma_{\rm IA}}{d\Omega_{k'}dE_{k'}d\Omega_{p'}dE_{p'}} \propto \sigma_{ep}\, S(\mathbf{p},E)\, T_A(E_{p'})$$

spectral function

T. de Forest Jr., NPA 392, 232 (1983)

Mean-field part of the spectral function

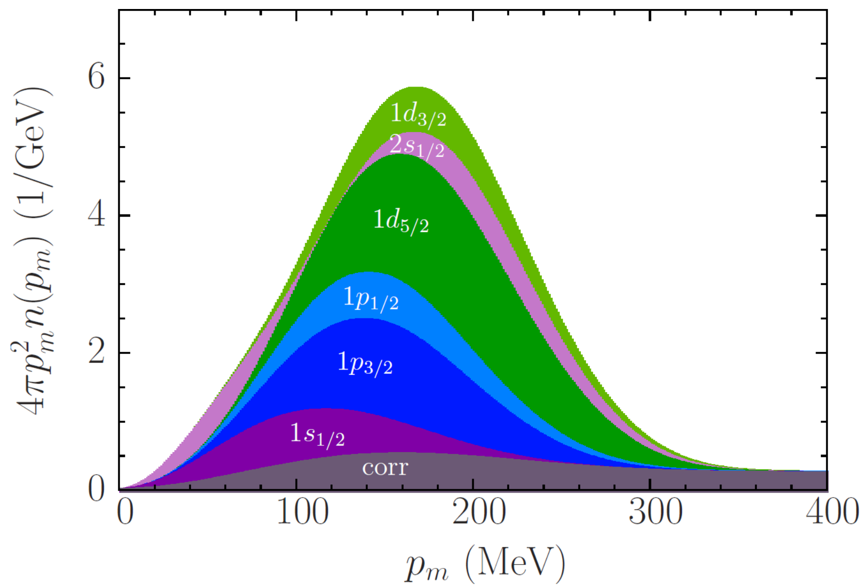
spectroscopic factor

energy distribution

$$P_{\mathrm{MF}}(p_m, E_m) = \sum_{\alpha} S_\alpha |\phi_\alpha(p_m)|^2 f_\alpha(E_m)$$

wave function in momentum space

Relativistic MF calculations by C. Giusti

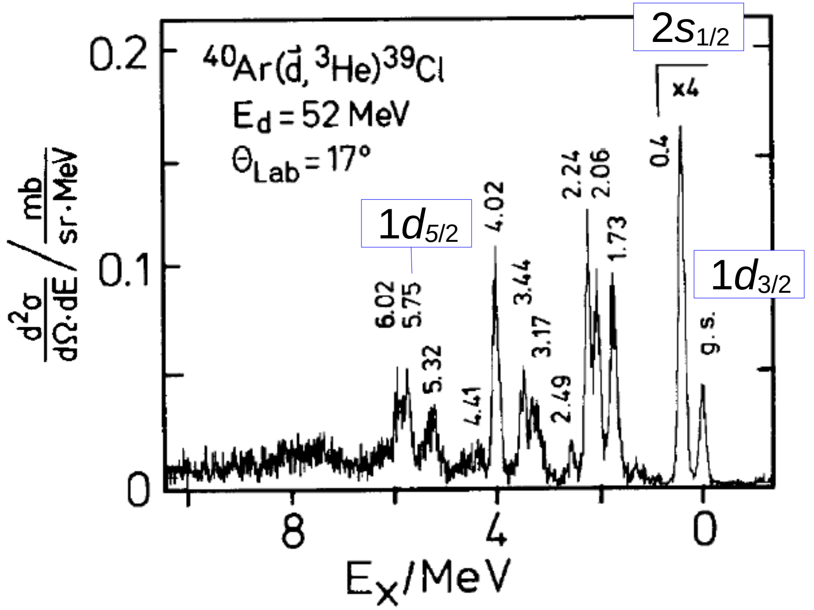


Mean-field part of the spectral function

α	S_α	E_α (MeV)
$1d_{3/2}$	1.6	12.53
$2s_{1/2}$	1.6	12.93
$1d_{5/2}$	4.8	18.23

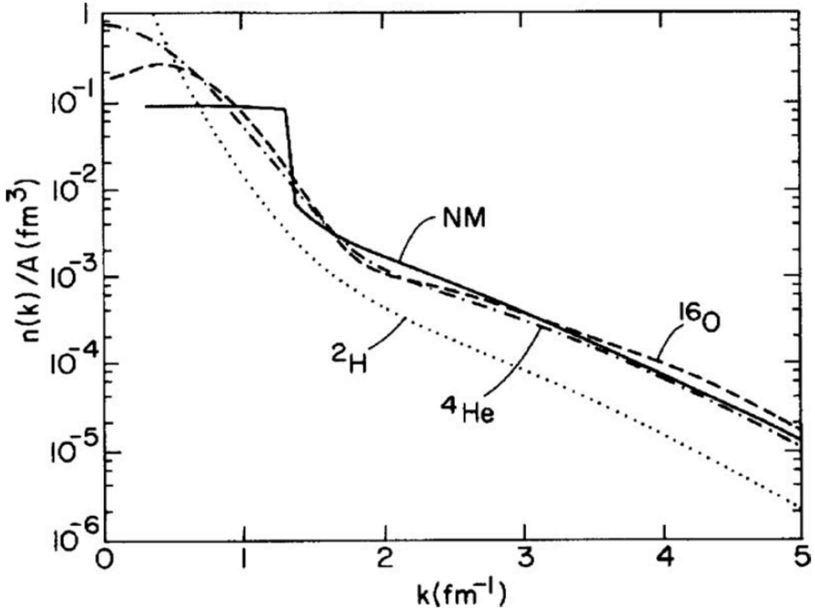
- $1d_{3/2}$: from the mass difference between ^{40}Ar and ^{39}Cl + p + e
- $2s_{1/2}$ and $1d_{5/2}$: from the dominant contribs. in the past ^{40}Ar(d, ^{3}He)^{39}Cl measurements
- Lower levels were not probed with deuteron
- Assumed Maxwell-Boltzmann distribution of missing energy

Mairle *et al.*, NPA **565**, 543 (1993)



Correlated part of the spectral function

Benhar *et al.,* RMP **80**, 189 (2008)



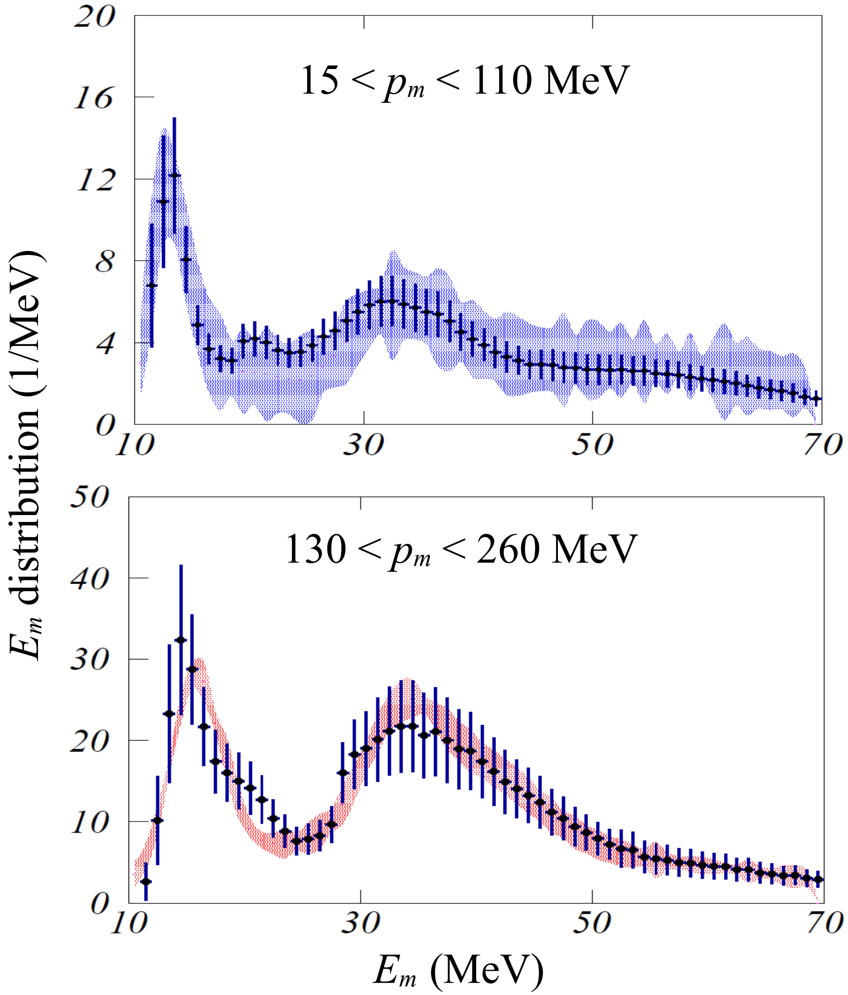
Ciofi degli Atti and Simula, PRC **53**, 1689 (1996)

- Correlated nucleons form quasi-deuteron pairs, with the relative momentum distributed as in deuteron.
- *NN* pairs undergo CM motion (Gaussian distrib.)
- Excitation energy of the (A − 1)-nucleons is their kinetic energy plus the *pn* knockout threshold

Missing energy distributions for Ar and Ti

α	E_α (MeV) w/ priors	E_α (MeV) w/o priors	σ_α (MeV) w/ priors	σ_α (MeV) w/o priors
$1d_{3/2}$	12.53 ± 0.02	10.90 ± 0.12	1.9 ± 0.4	1.6 ± 0.4
$2s_{1/2}$	12.92 ± 0.02	12.57 ± 0.38	3.8 ± 0.8	3.0 ± 1.8
$1d_{5/2}$	18.23 ± 0.02	17.77 ± 0.80	9.2 ± 0.9	9.6 ± 1.3
$1p_{1/2}$	28.8 ± 0.7	28.7 ± 0.7	12.1 ± 1.0	12.0 ± 3.6
$1p_{3/2}$	33.0 ± 0.3	33.0 ± 0.3	9.3 ± 0.5	9.3 ± 0.5
$1s_{1/2}$	53.4 ± 1.1	53.4 ± 1.0	28.3 ± 2.2	28.1 ± 2.3
corr.	24.1 ± 2.7	24.1 ± 1.7	—	—

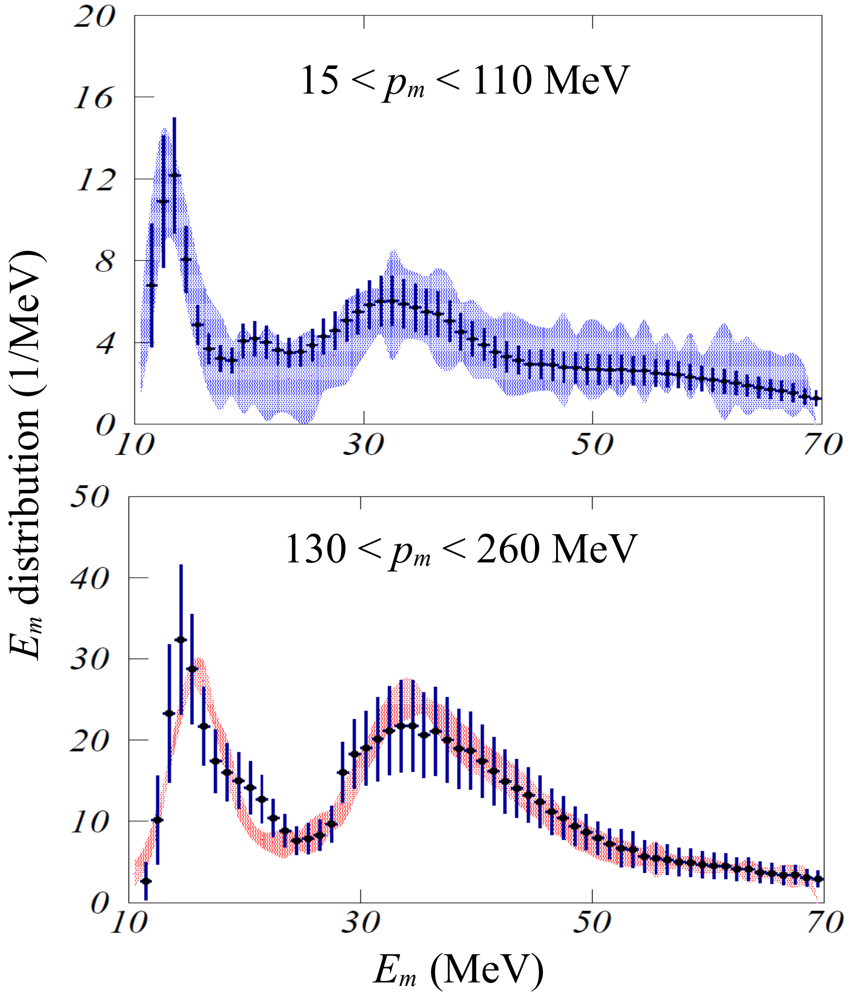
α	E_α (MeV) w/ priors	E_α (MeV) w/o priors	σ_α (MeV) w/ priors	σ_α (MeV) w/o priors
$1f_{7/2}$	11.32 ± 0.10	11.31 ± 0.10	8.00 ± 5.57	8.00 ± 6.50
$1d_{3/2}$	12.30 ± 0.24	12.33 ± 0.24	7.00 ± 0.61	7.00 ± 3.84
$2s_{1/2}$	12.77 ± 0.25	12.76 ± 0.25	7.00 ± 3.76	7.00 ± 3.84
$1d_{5/2}$	15.86 ± 0.20	15.91 ± 0.22	2.17 ± 0.27	2.23 ± 0.29
$1p_{1/2}$	33.33 ± 0.60	33.15 ± 0.65	3.17 ± 0.45	3.03 ± 0.48
$1p_{3/2}$	39.69 ± 0.62	39.43 ± 0.68	5.52 ± 0.70	5.59 ± 0.70
$1s_{1/2}$	53.84 ± 1.86	52.00 ± 3.13	11.63 ± 1.90	13.63 ± 2.59
corr.	25.20 ± 0.02	25.00 ± 0.29	—	—



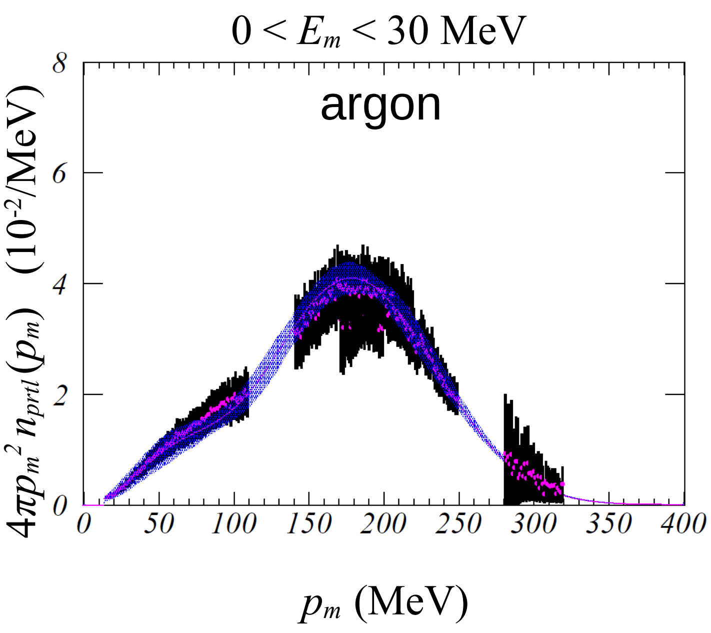
Spectroscopic factors for Ar and Ti

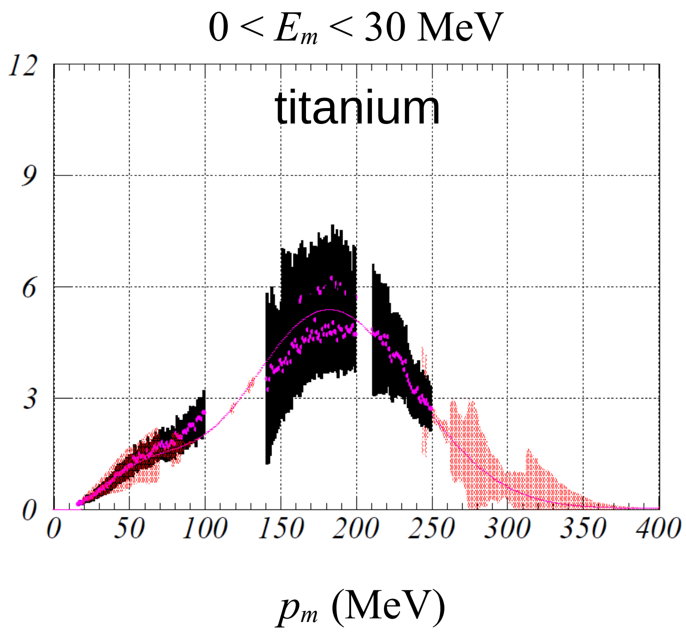
		all priors	w/o p_m	w/o corr.
α	N_α		S_α	
$1d_{3/2}$	2	0.89 ± 0.11	1.42 ± 0.20	0.95 ± 0.11
$2s_{1/2}$	2	1.72 ± 0.15	1.22 ± 0.12	1.80 ± 0.16
$1d_{5/2}$	6	3.52 ± 0.26	3.83 ± 0.30	3.89 ± 0.30
$1p_{1/2}$	2	1.53 ± 0.21	2.01 ± 0.22	1.83 ± 0.21
$1p_{3/2}$	4	3.07 ± 0.05	2.23 ± 0.12	3.12 ± 0.05
$1s_{1/2}$	2	2.51 ± 0.05	2.05 ± 0.23	2.52 ± 0.05
corr.	0	3.77 ± 0.28	3.85 ± 0.25	excluded
$\sum_\alpha S_\alpha$		17.02 ± 0.48	16.61 ± 0.57	14.12 ± 0.42
d.o.f		206	231	232
χ^2/d.o.f.		1.9	1.4	2.0

$1f_{7/2}$	2	1.53 ± 0.25	1.55 ± 0.28	1.24 ± 0.22
$1d_{3/2}$	4	2.79 ± 0.37	3.15 ± 0.54	3.21 ± 0.37
$2s_{1/2}$	2	2.00 ± 0.11	1.78 ± 0.46	2.03 ± 0.11
$1d_{5/2}$	6	2.25 ± 0.16	2.34 ± 0.19	3.57 ± 0.29
$1p_{1/2}$	2	2.00 ± 0.20	1.80 ± 0.27	2.09 ± 0.19
$1p_{3/2}$	4	2.90 ± 0.20	2.92 ± 0.20	4.07 ± 0.15
$1s_{1/2}$	2	2.14 ± 0.10	2.56 ± 0.30	2.14 ± 0.11
corr.	0	4.71 ± 0.31	4.21 ± 0.46	excluded
$\sum_\alpha S_\alpha$		20.32 ± 0.65	20.30 ± 1.03	18.33 ± 0.59
d.o.f		121	153	125
χ^2/d.o.f.		0.95	0.71	1.23



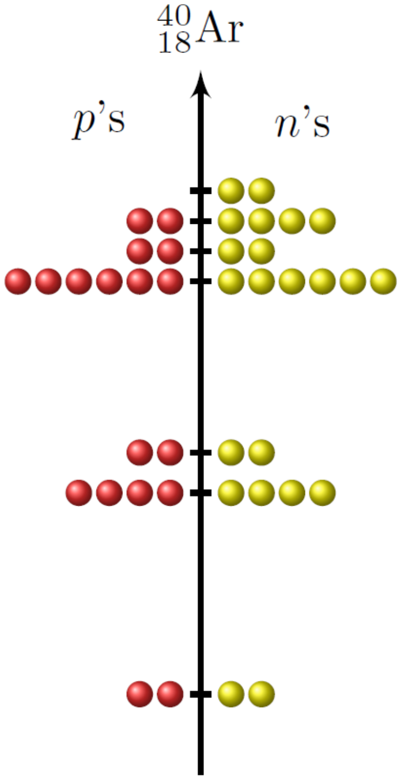
Partial momentum distributions





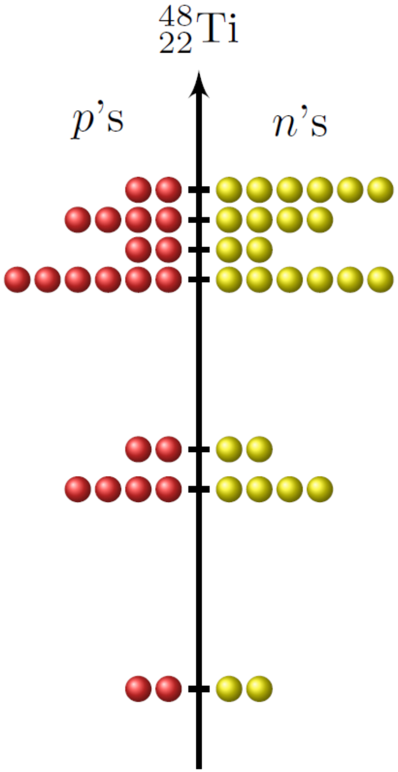
Data from different kinematics are consistent within uncertainties.

Energy levels

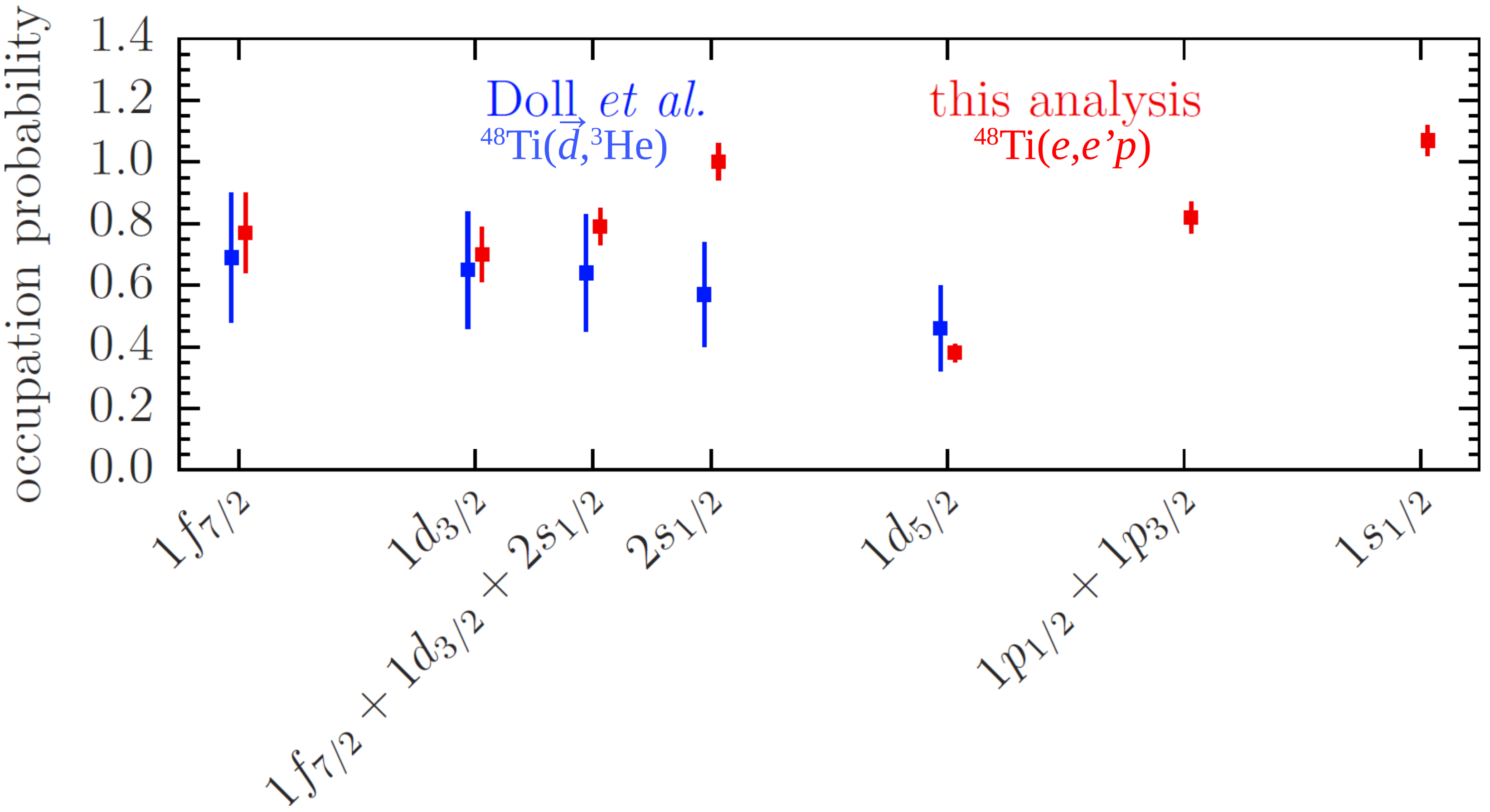


^{40}Ar		^{48}Ti
neutrons		protons
9.87	1f7/2	11.45
11.39	1d3/2	12.21
12.23	2s1/2	12.84
13.23	1d5/2	15.45

Agreement to 0.6–2.2 MeV

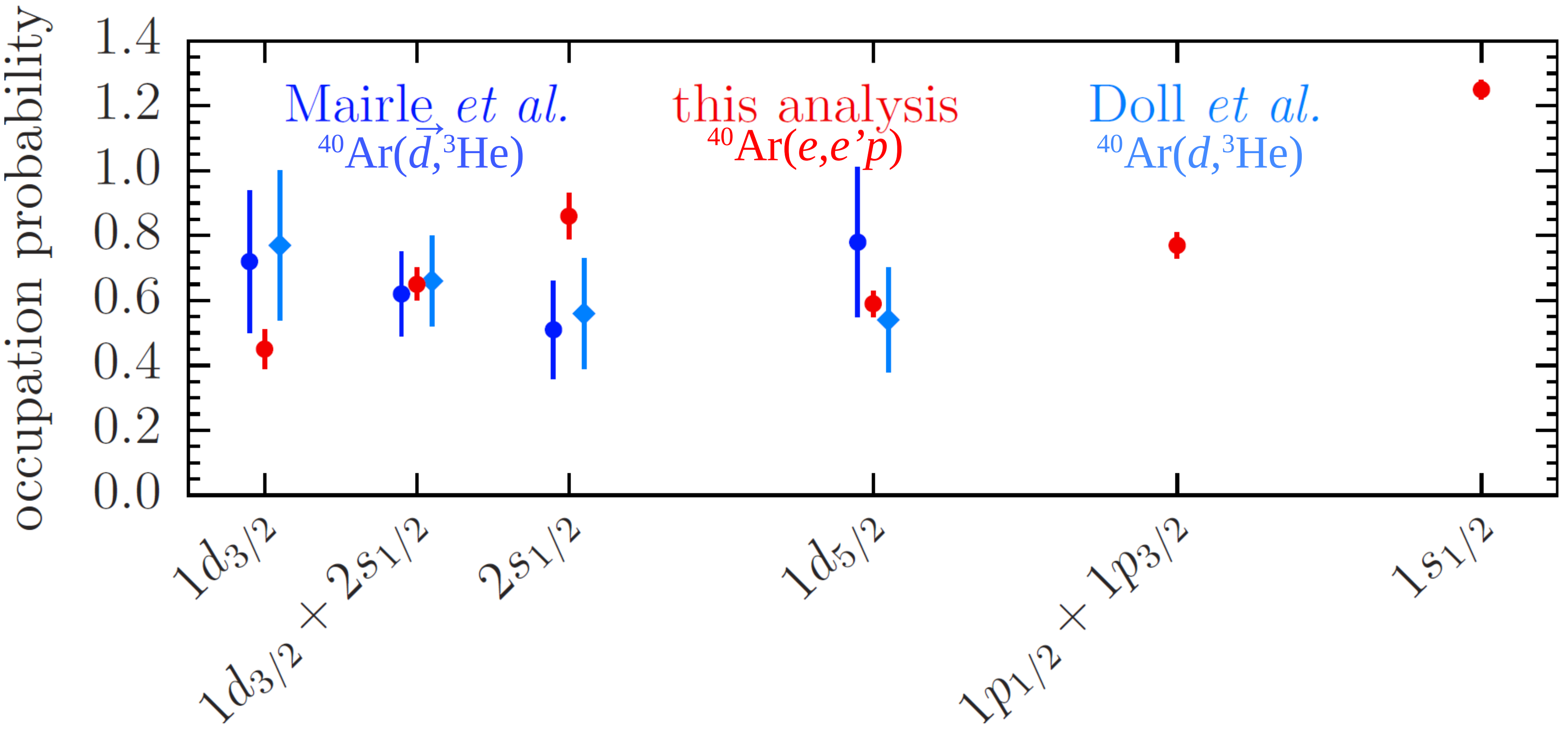


Occupation probability



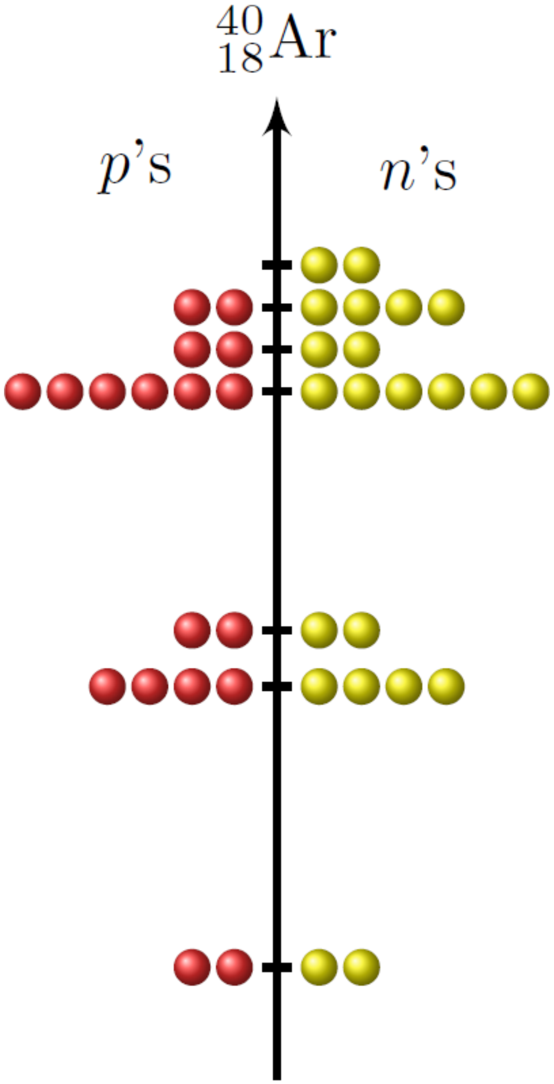
52-MeV polarized [Doll *et al.*, JPG **5**, 1421 (1979); E_x < 7.54 MeV] deuteron beam at Karlsruhe

Occupation probability



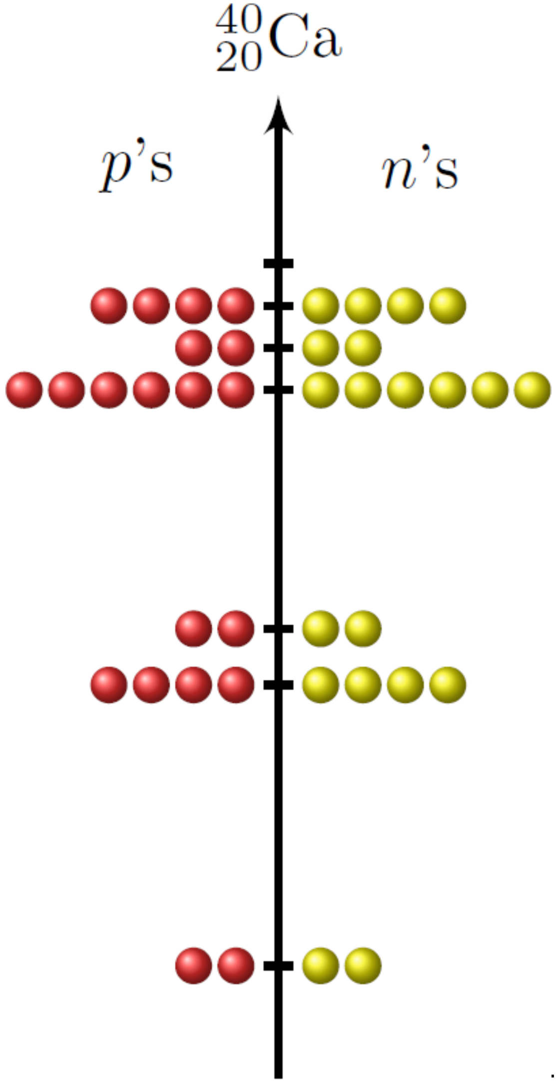
52-MeV polarized [Mairle *et al.*, NPA **565**, 543 (1993); E_x < 9 MeV] and unpolarized [Doll *et al.*, NPA **230**, 329 (1974); **129**, 469 (1969); E_x < 7 MeV] deuteron beam at Karlsruhe

Kramer *et al.* [NPA **679**, 267 (2001)]: reanalysis of (*d*,^{3}He) experiments, $S_\alpha \rightarrow S_\alpha/1.5$



proton energy levels

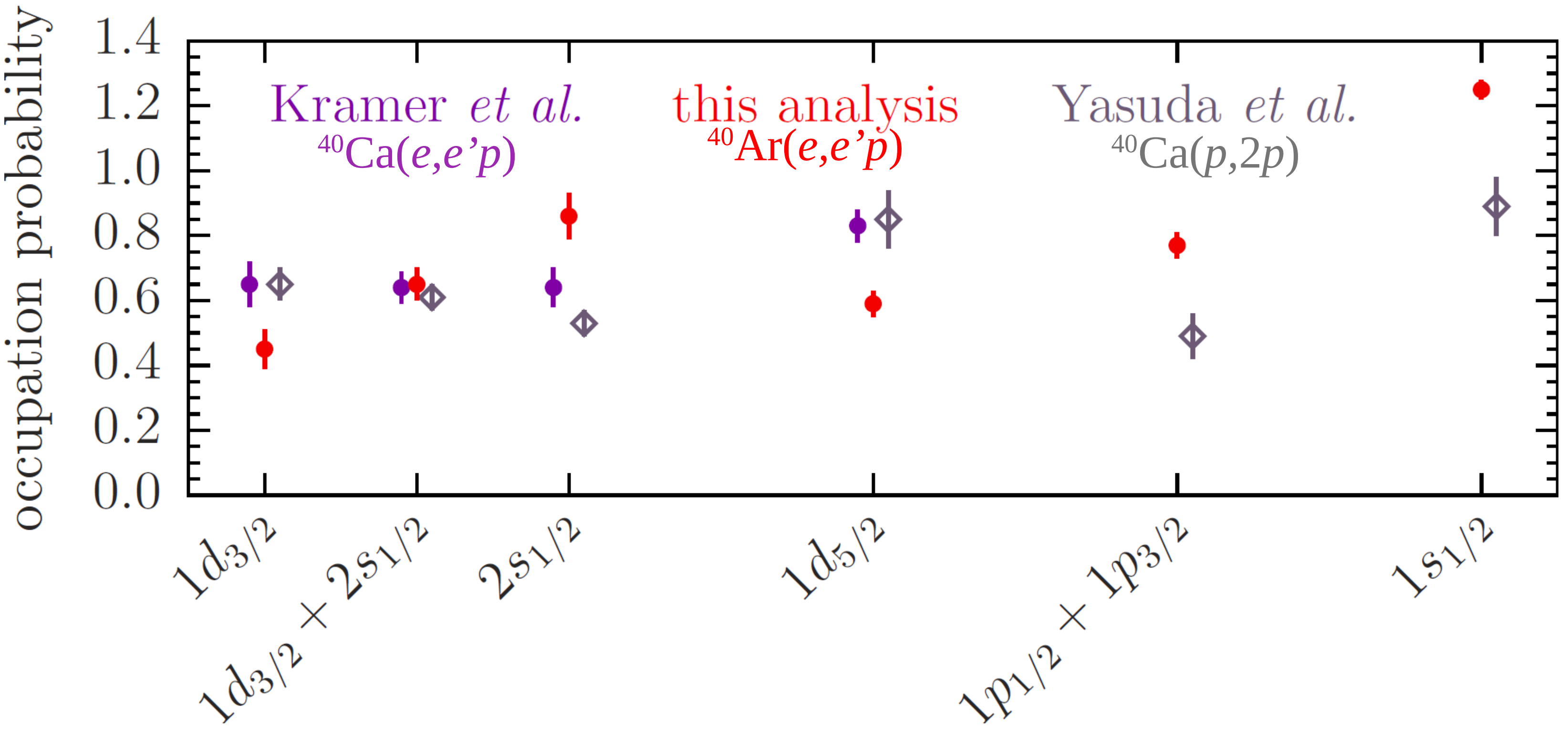
Ar		Ca
12.53(2)	1d3/2	8.5(1)
12.92(2)	2s1/2	11.0(1)
18.23(2)	1d5/2	15.7(1)
28.8(7)	1p1/2	29.8(7)
33.0(3)	1p3/2	34.7(3)
53.4(1.1)	1s1/2	53.6(7)



Jiang *et al.*,
PRD 105, 112002 (2022)

Volkov *et al.*
SJNP 52, 848 (1990)

Occupation probability



Kramer *et al.* [Ph.D. thesis (1990)]: ~340–440-MeV electron beam at NIKHEF-K

Yasuda *et al.* [Ph.D. thesis (2012)]: 392-MeV polarized proton beam at RCNP

Directions for future improvements

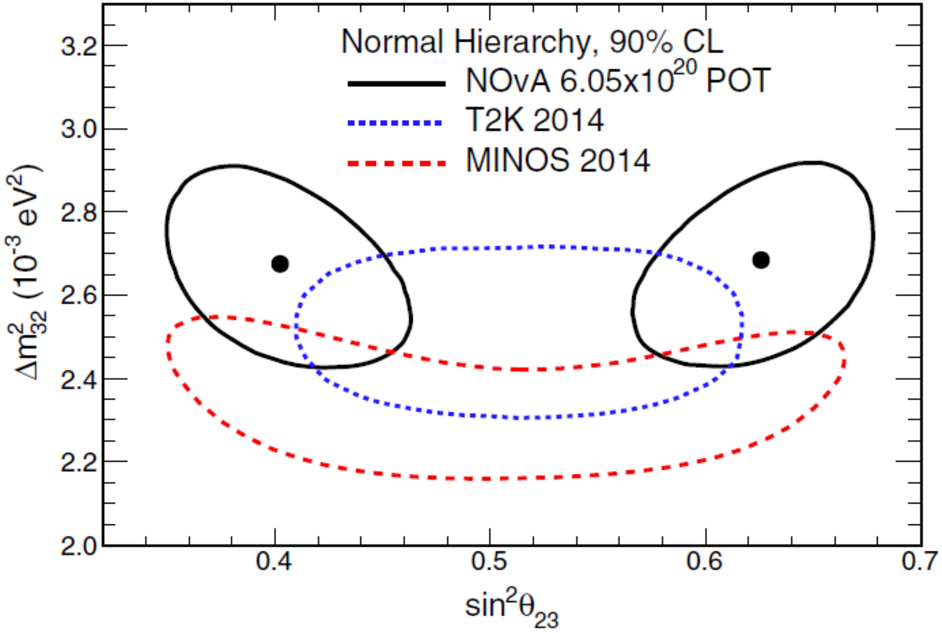
- 2D analysis
- Final-state interactions
- Wave functions
- Correlated part of the spectral function

Summary

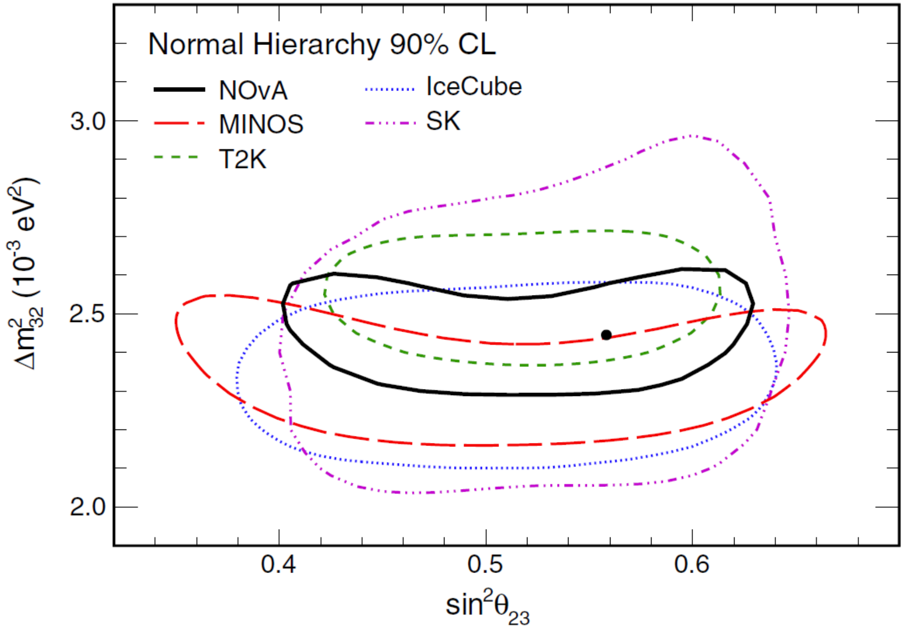
- The success of the long-baseline neutrino program requires reliable cross sections.
- The spectral function approach is a viable option.
- The first, exploratory analysis of the full dataset of the JLab experiment E12-14-012 found reasonable parametrizations of the spectral functions of ^{40}Ar and ^{48}Ti.
- Comparison with past results shows strengths and limitations.
- Separation of individual contributions requires improved analysis. Numerous theoretical developments are necessary.


Thank you!

Concrete example: NOvA



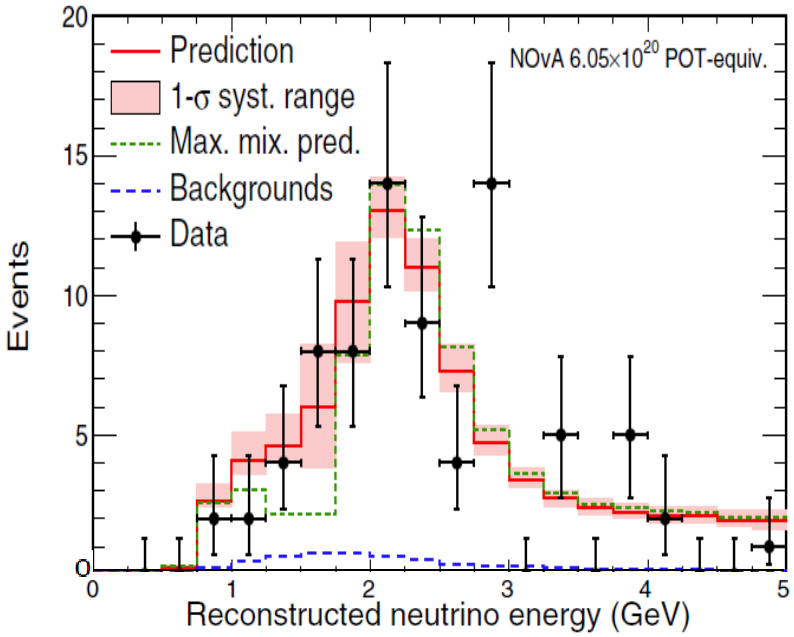
Acero *et al.* (NOvA), PRL 118, 151802 (2017)



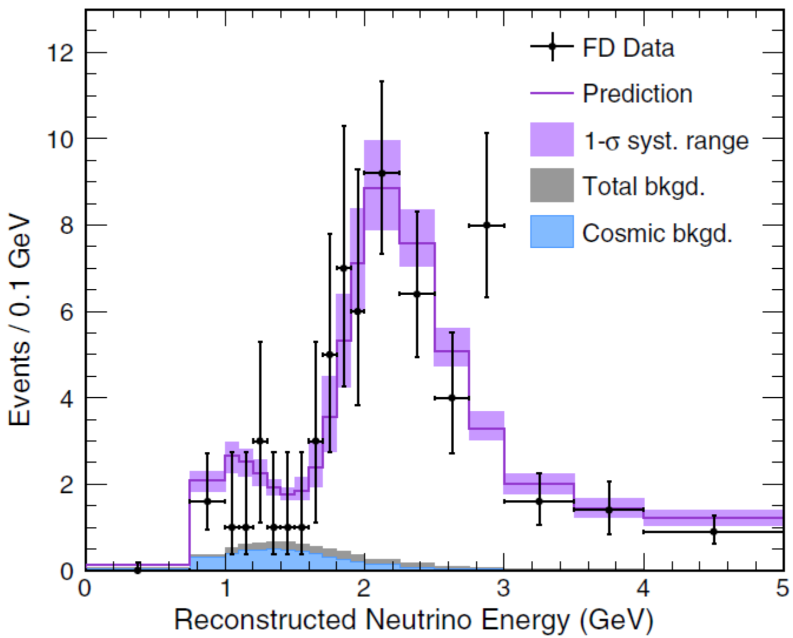
Acero *et al.* (NOvA), PRD 98, 032012 (2018)

"This change was caused by three changes ... The largest effect was due to new simulations and calibrations."

Concrete example: NOvA



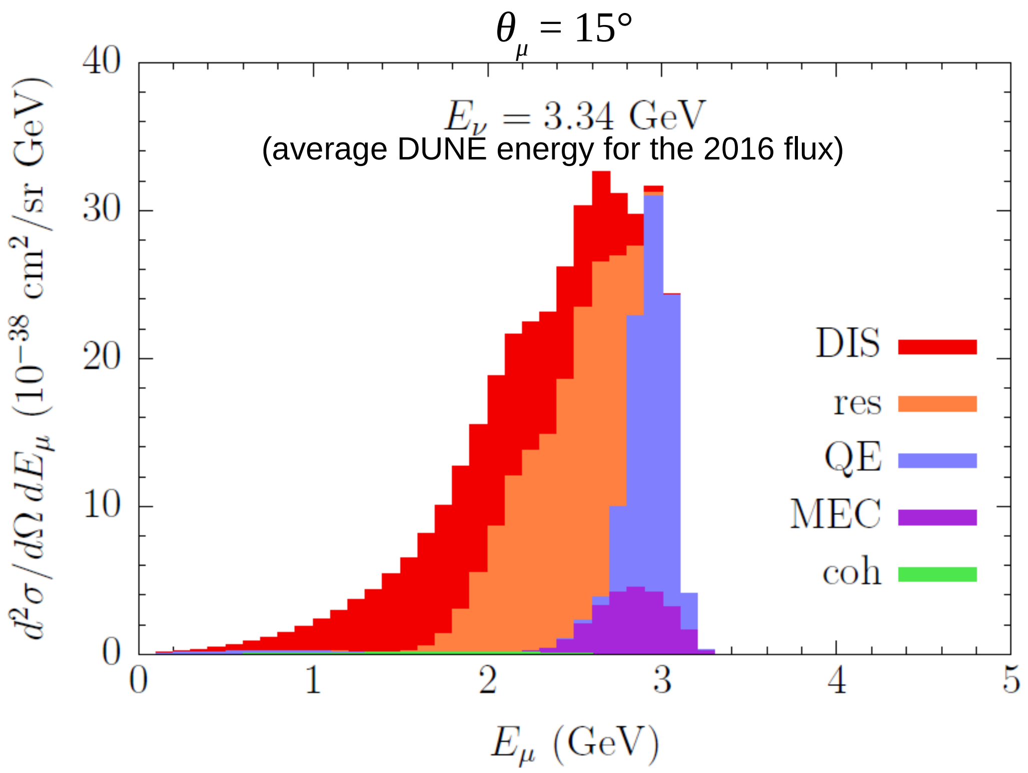
Acero *et al.* (NOvA), PRL 118, 151802 (2017)



Acero *et al.* (NOvA), PRD 98, 032012 (2018)

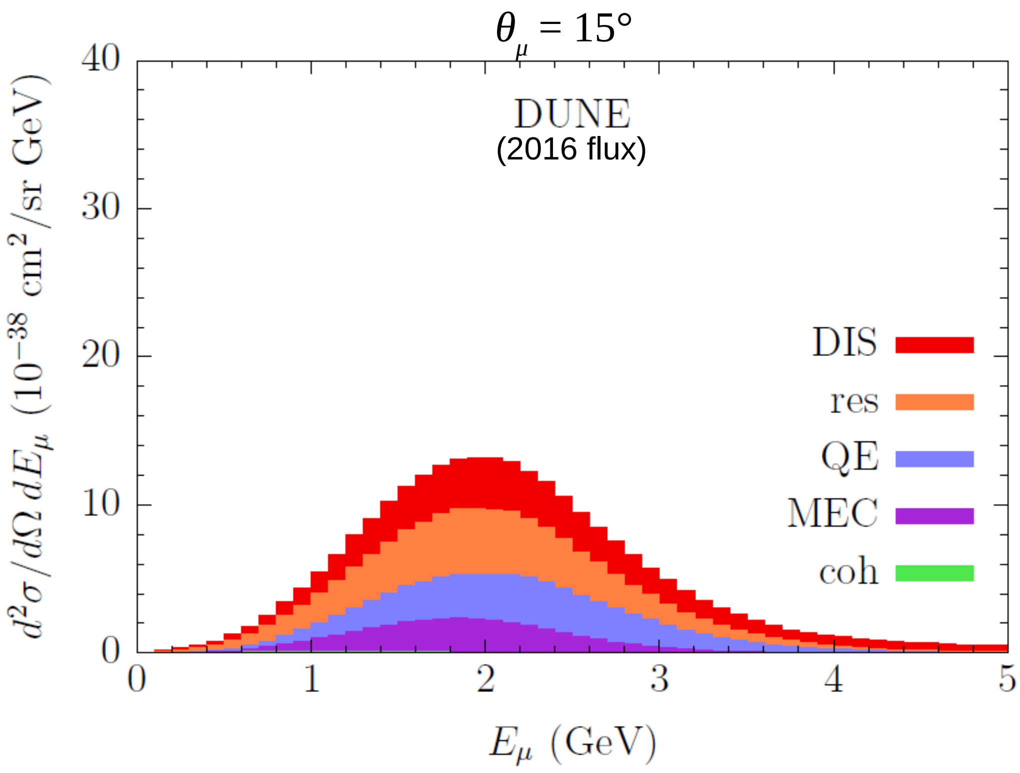
"This change was caused by three changes ... The largest effect was due to new simulations and calibrations."

Neutrino double differential cross section



A.M.A. & A. Friedland, PRD 102, 053001 (2020)

Neutrino double differential cross section



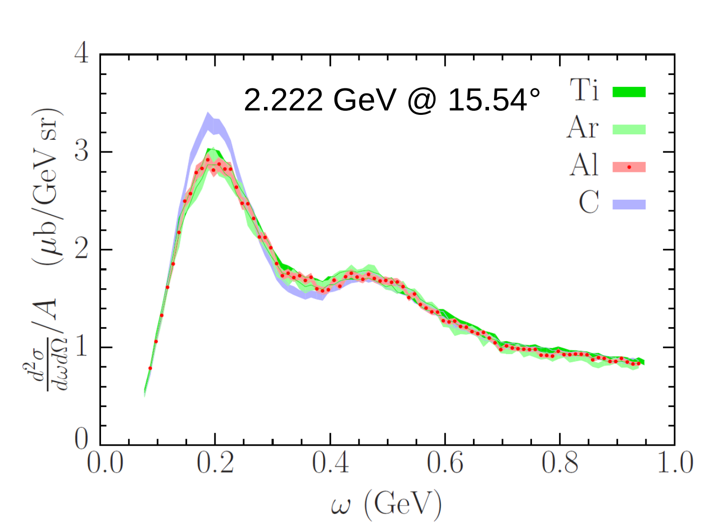
A.M.A. & A. Friedland, PRD 102, 053001 (2020)

Previous results

- Inclusive cross sections for C and Ti
[Dai *et al.*, PRC 98, 014617 (2018)]

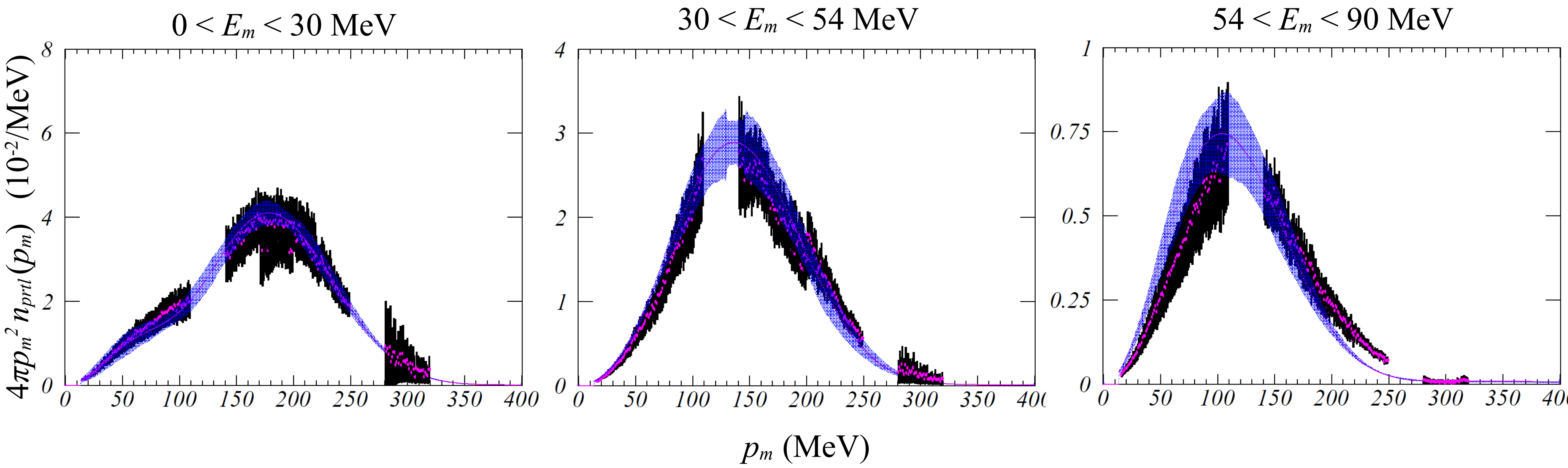
- Inclusive cross section for Ar
[Dai *et al.*, PRC 99, 054608 (2019)]

- Inclusive cross section for Al-7075,
A-, *y*- ,*ψ*-scaling of all (*e*,*e*') data
[Murphy *et al.*, PRC 100, 054606 (2019)]



- Exclusive Ar & Ti cross sections for a single kinematics, p_m ~ 50–60 MeV, E_m ~ 50–70 MeV [Gu *et al.*, PRC 103, 034604 (2021)]

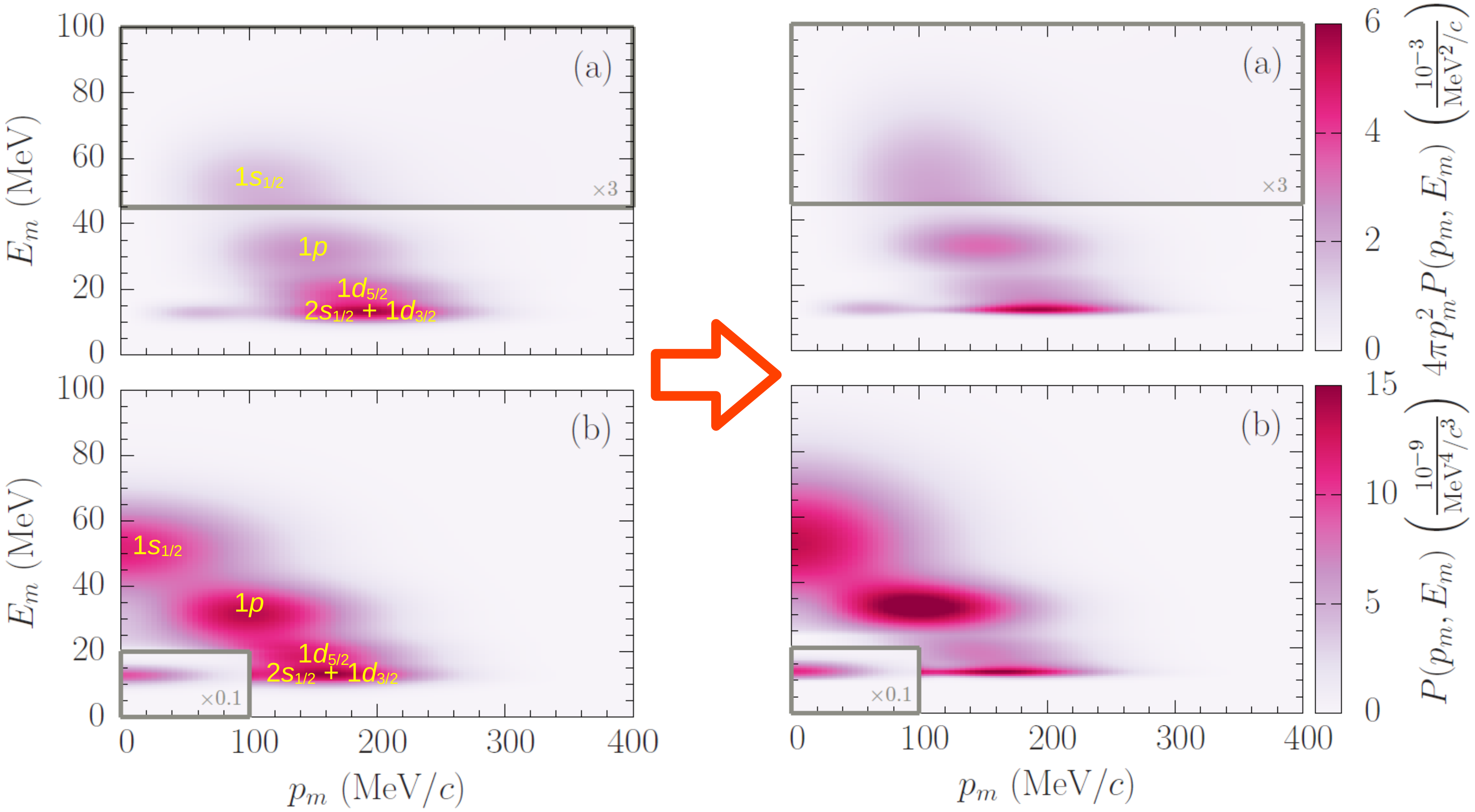
Partial momentum distributions



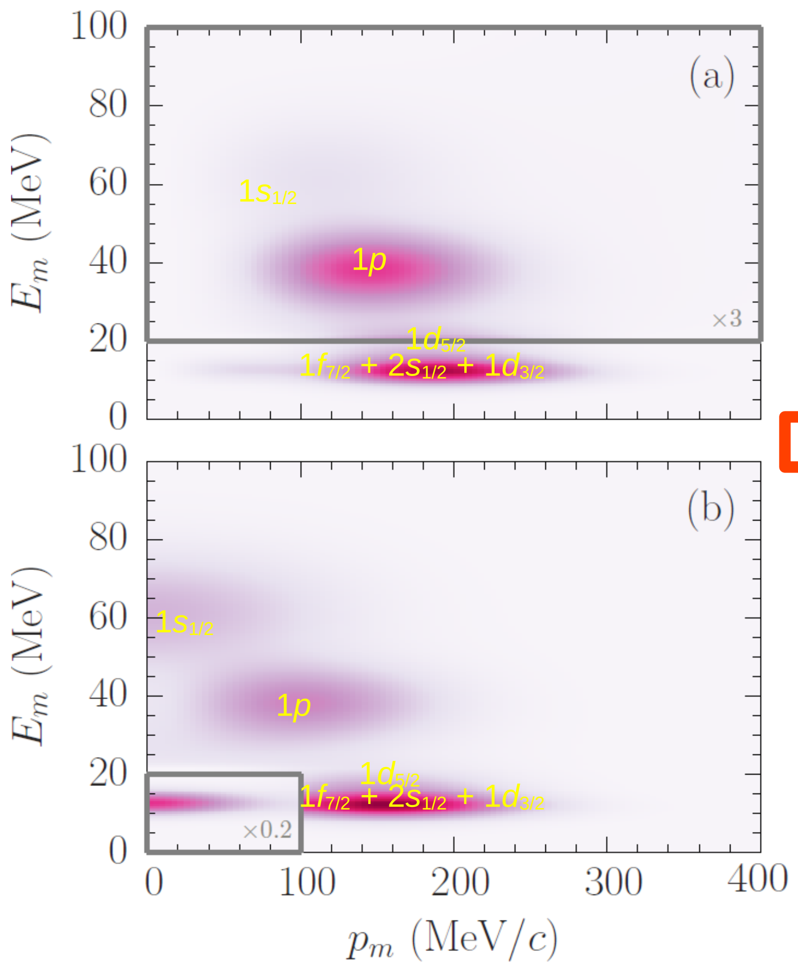
Data from different kinematics are consistent within uncertainties.

Test spectral function

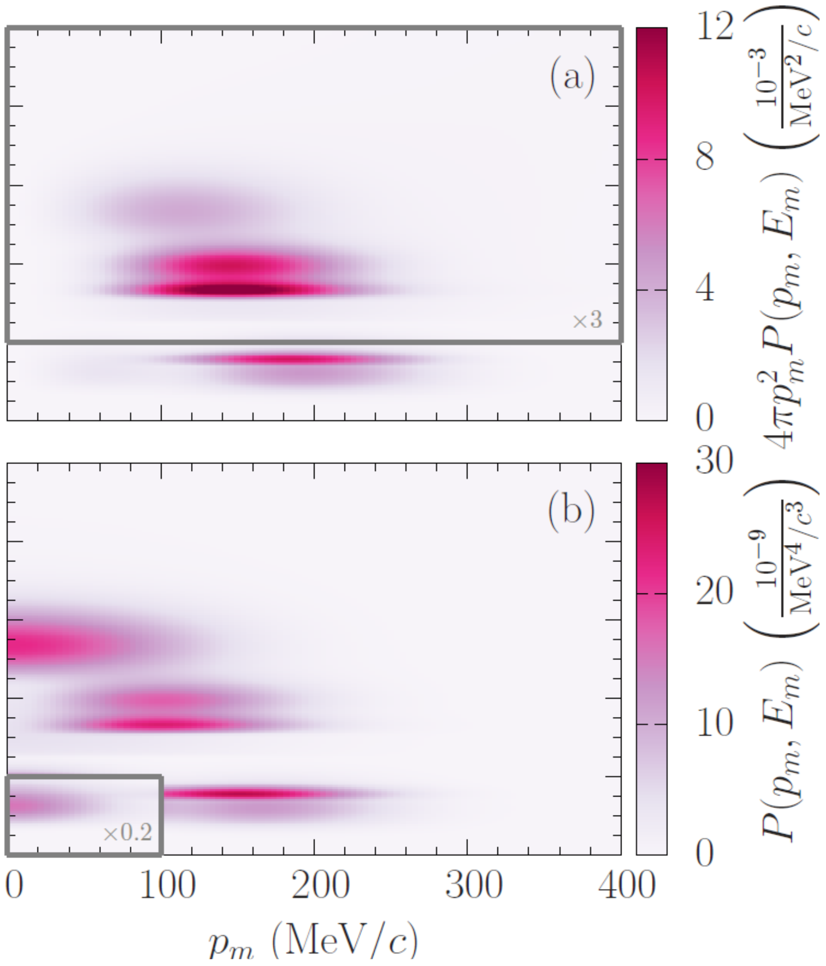
Extracted spectral function



Test spectral function



Extracted spectral function



p_m fit results for Ti

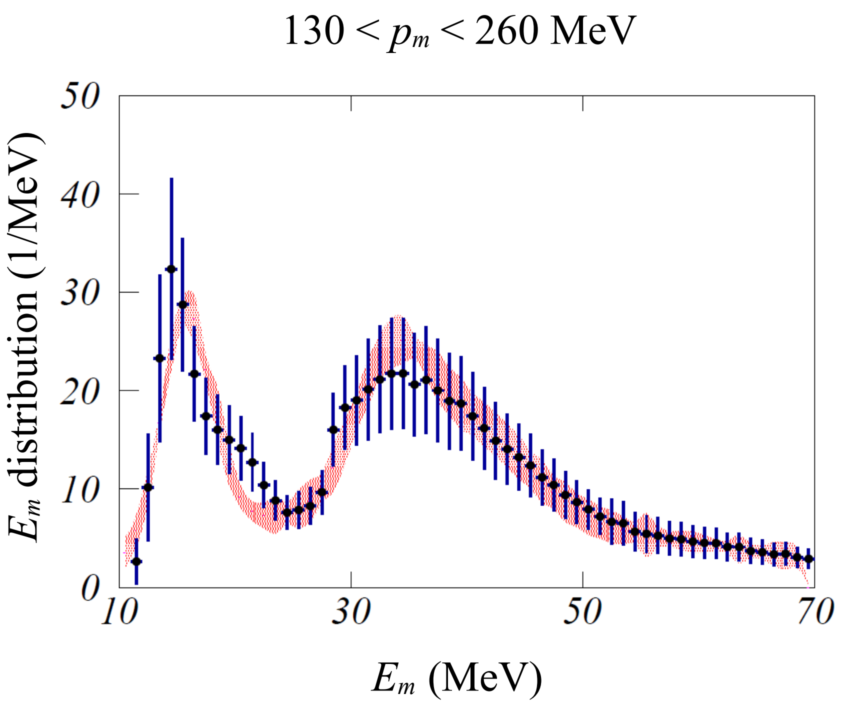
		w/ corr.	w/o corr.
α	N_α	S_α	
$1f_{7/2}$	2	0.83 ± 1.17	0.78 ± 1.35
$1d_{3/2}$	4	1.17 ± 0.22	1.34 ± 0.10
$2s_{1/2}$	2	2.02 ± 0.08	2.18 ± 0.08
$1d_{5/2}$	6	2.34 ± 1.34	2.34 ± 3.72
$1p_{1/2}$	2	2.46 ± 0.27	2.71 ± 1.19
$1p_{3/2}$	4	5.46 ± 1.69	5.46 ± 0.05
$1s_{1/2}$	2	2.17 ± 0.09	2.51 ± 0.08
corr.	0	5.15 ± 0.41	excluded
$\sum_\alpha S_\alpha$		21.60 ± 2.51	17.32 ± 4.20
d.o.f.		675	676
χ^2/d.o.f.		0.49	0.57

In the p_m fit, only deeply bound states are sensitive to the correlated spectral function.

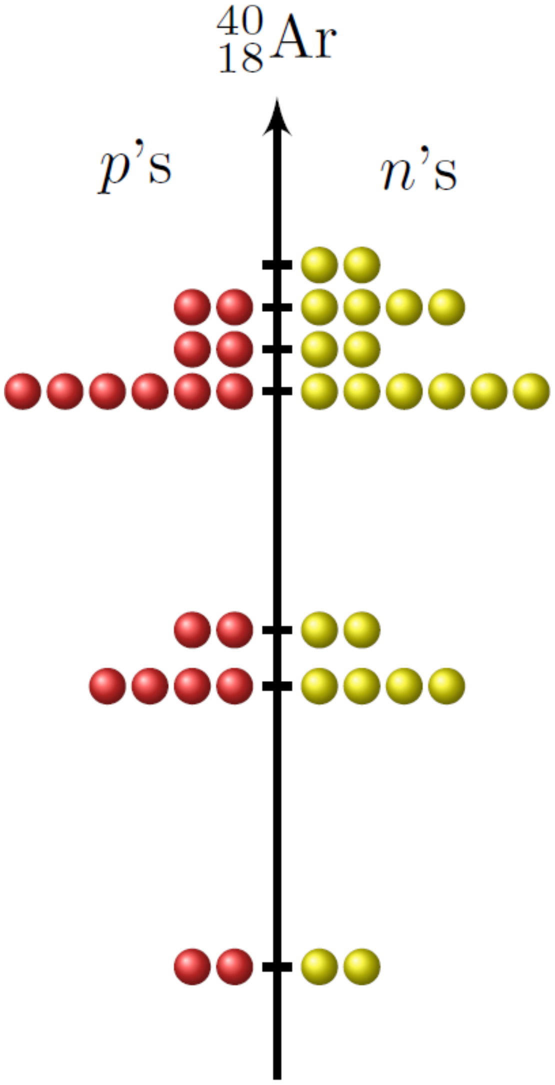
E_m fit results for Ti

α	N_α	all priors	w/o p_m	w/o corr.
		S_α		
$1f_{7/2}$	2	1.53 ± 0.25	1.55 ± 0.28	1.24 ± 0.22
$1d_{3/2}$	4	2.79 ± 0.37	3.15 ± 0.54	3.21 ± 0.37
$2s_{1/2}$	2	2.00 ± 0.11	1.78 ± 0.46	2.03 ± 0.11
$1d_{5/2}$	6	2.25 ± 0.16	2.34 ± 0.19	3.57 ± 0.29
$1p_{1/2}$	2	2.00 ± 0.20	1.80 ± 0.27	2.09 ± 0.19
$1p_{3/2}$	4	2.90 ± 0.20	2.92 ± 0.20	4.07 ± 0.15
$1s_{1/2}$	2	2.14 ± 0.10	2.56 ± 0.30	2.14 ± 0.11
corr.	0	4.71 ± 0.31	4.21 ± 0.46	excluded
$\sum_\alpha S_\alpha$		20.32 ± 0.65	20.30 ± 1.03	18.33 ± 0.59
d.o.f		121	153	125
χ^2/d.o.f.		0.95	0.71	1.23

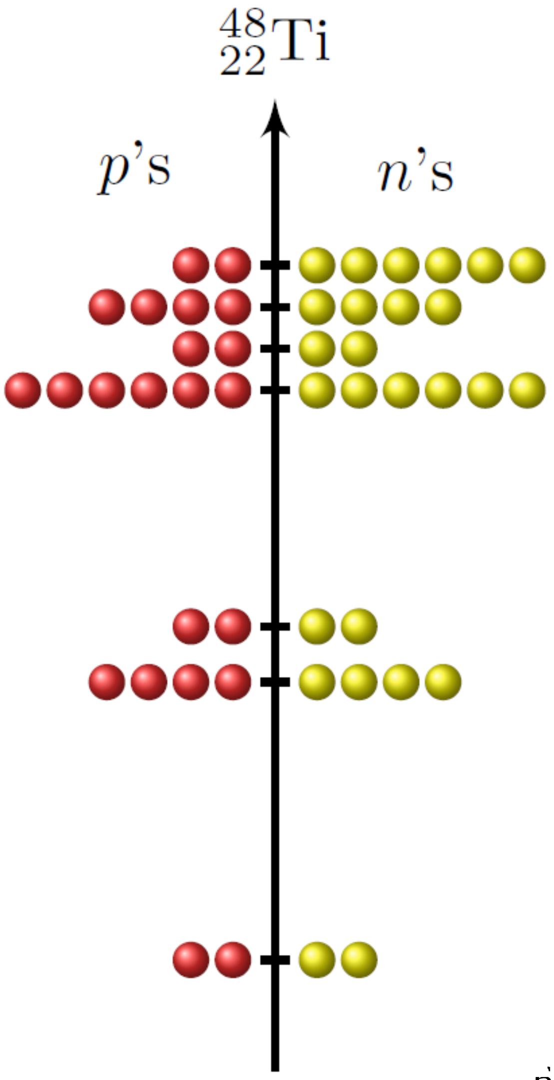
α	E_α (MeV) w/ priors	E_α (MeV) w/o priors	σ_α (MeV) w/ priors	σ_α (MeV) w/o priors
$1f_{7/2}$	11.32 ± 0.10	11.31 ± 0.10	8.00 ± 5.57	8.00 ± 6.50
$1d_{3/2}$	12.30 ± 0.24	12.33 ± 0.24	7.00 ± 0.61	7.00 ± 3.84
$2s_{1/2}$	12.77 ± 0.25	12.76 ± 0.25	7.00 ± 3.76	7.00 ± 3.84
$1d_{5/2}$	15.86 ± 0.20	15.91 ± 0.22	2.17 ± 0.27	2.23 ± 0.29
$1p_{1/2}$	33.33 ± 0.60	33.15 ± 0.65	3.17 ± 0.45	3.03 ± 0.48
$1p_{3/2}$	39.69 ± 0.62	39.43 ± 0.68	5.52 ± 0.70	5.59 ± 0.70
$1s_{1/2}$	53.84 ± 1.86	52.00 ± 3.13	11.63 ± 1.90	13.63 ± 2.59
corr.	25.20 ± 0.02	25.00 ± 0.29	—	—



proton energy levels



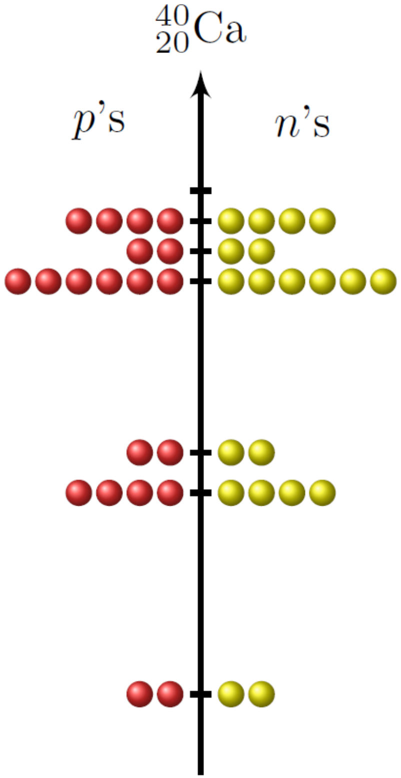
Ar		Ti
	1f7/2	11.32(10)
12.53(2)	1d3/2	12.30(24)
12.92(2)	2s1/2	12.77(25)
18.23(2)	1d5/2	15.86(20)
28.8(7)	1p1/2	33.3(6)
33.0(3)	1p3/2	39.7(6)
33.0(3)	1p3/2	39.7(6)
53.4(1.1)	1s1/2	53.8(1.9)



Jiang *et al.*,
PRD 105, 112002 (2022)

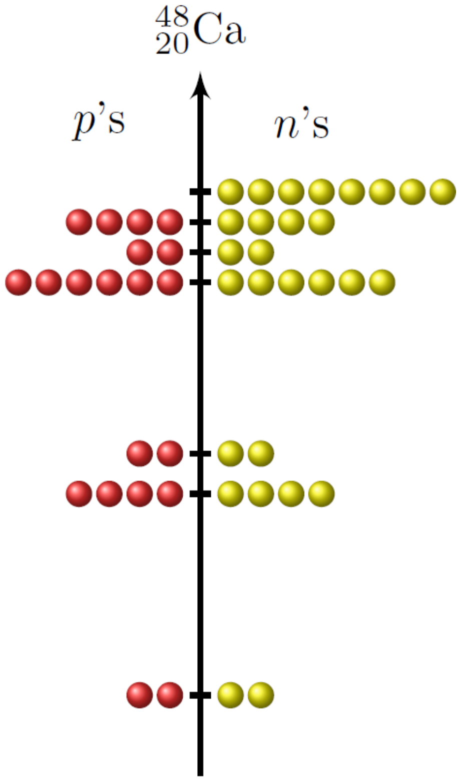
Jiang *et al.*
PRD 107, 012005 (2023)

Calcium isotopes



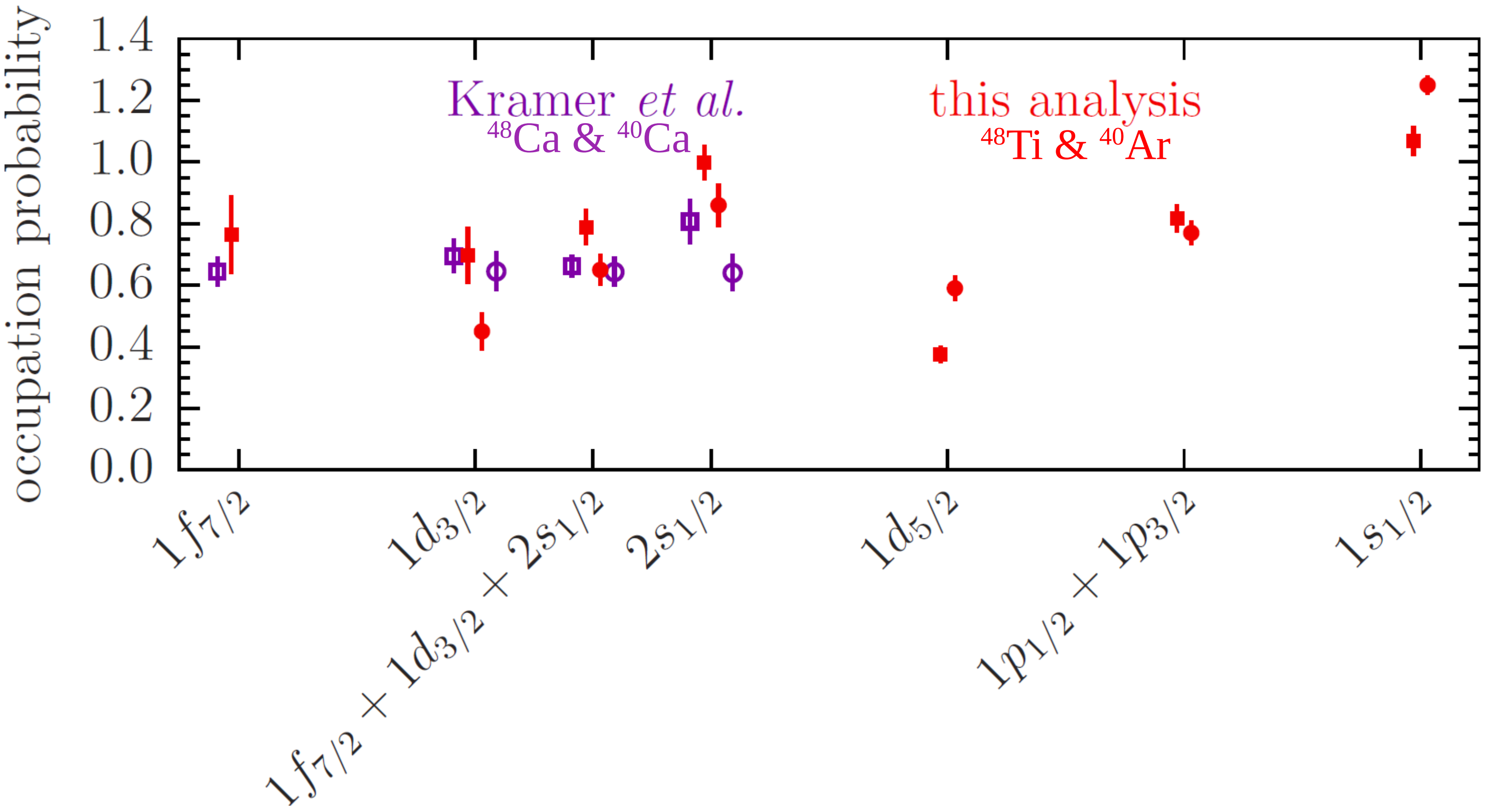
^{40}Ca		^{48}Ca
8.3(3)	1d3/2	16.8(3)
11.1(3)	2s1/2	17.1(3)
16.8(4)	1d5/2	23.9(7)

Kramer, Ph.D. thesis (1990)



6–8.5 MeV differences

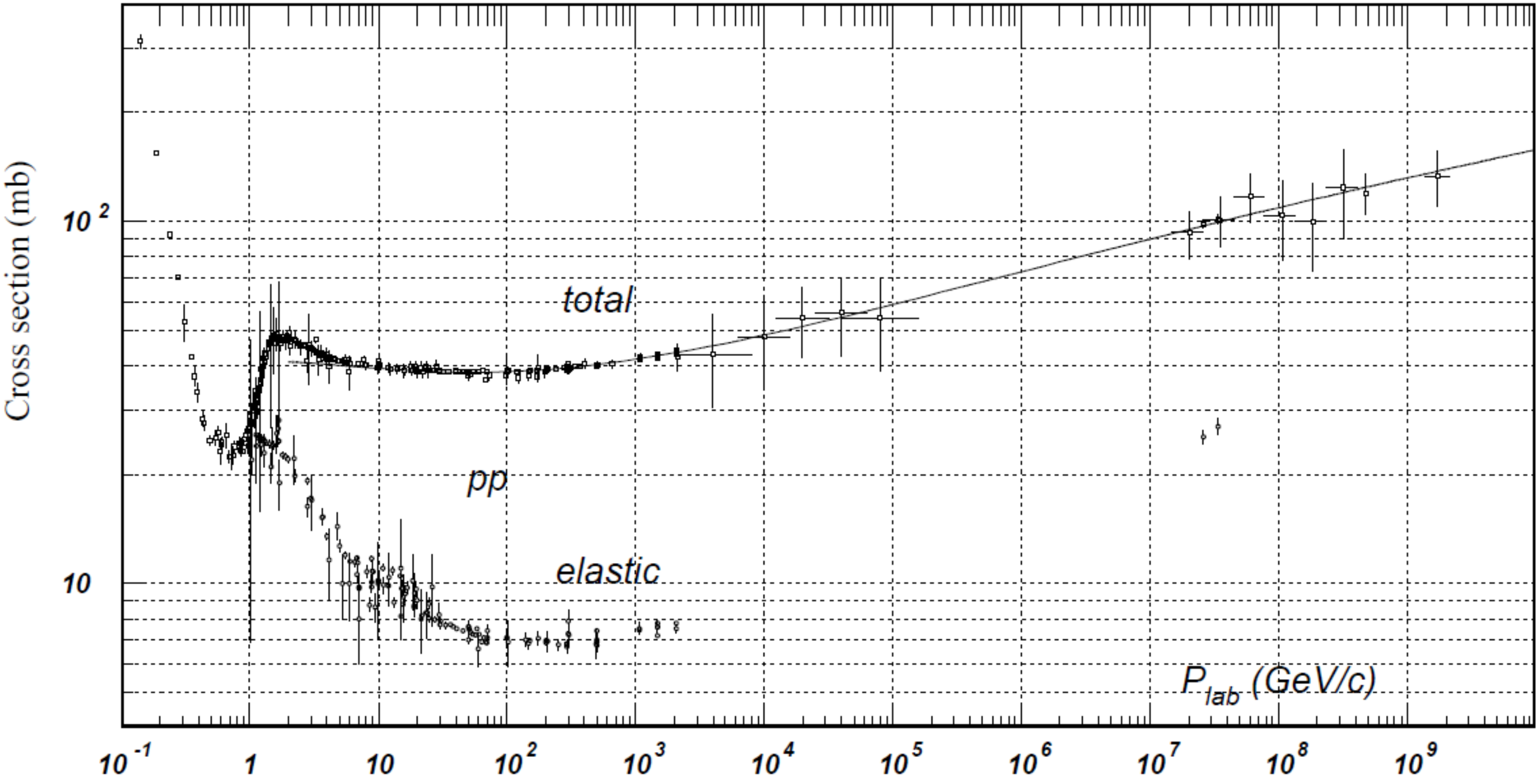
Occupation probability



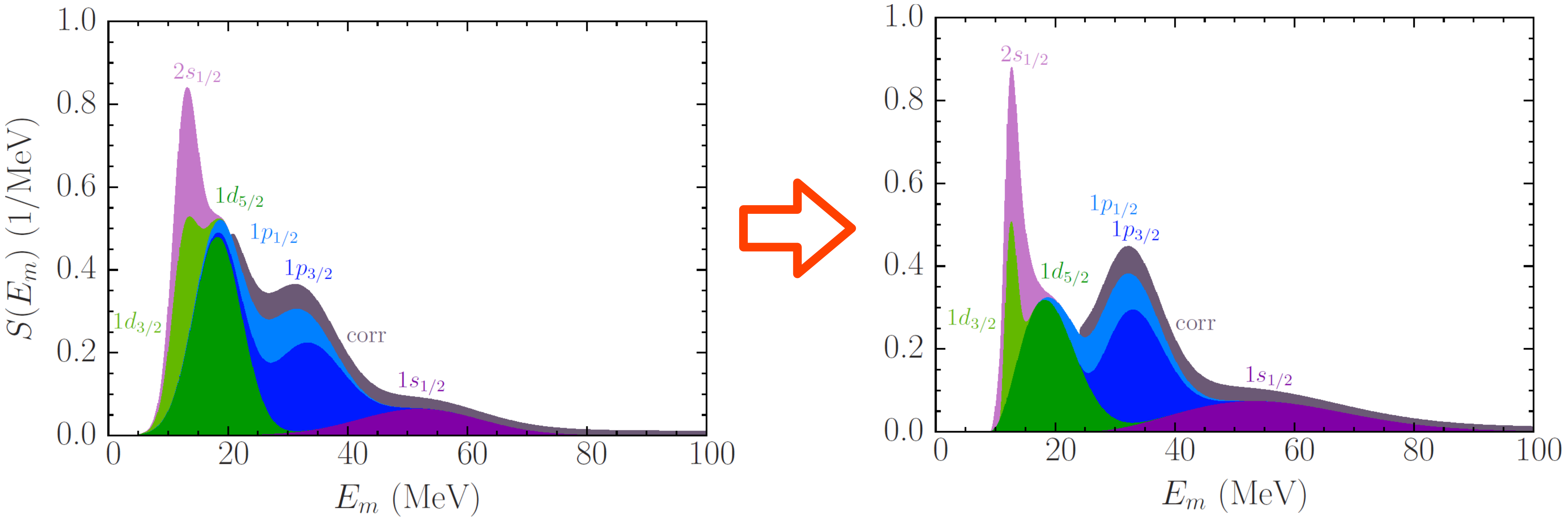
Kramer *et al.* [Ph.D. thesis (1990)]: ~340–440-MeV electron beam at NIKHEF-K

TABLE I. Kinematics settings used to collect the data analyzed here.

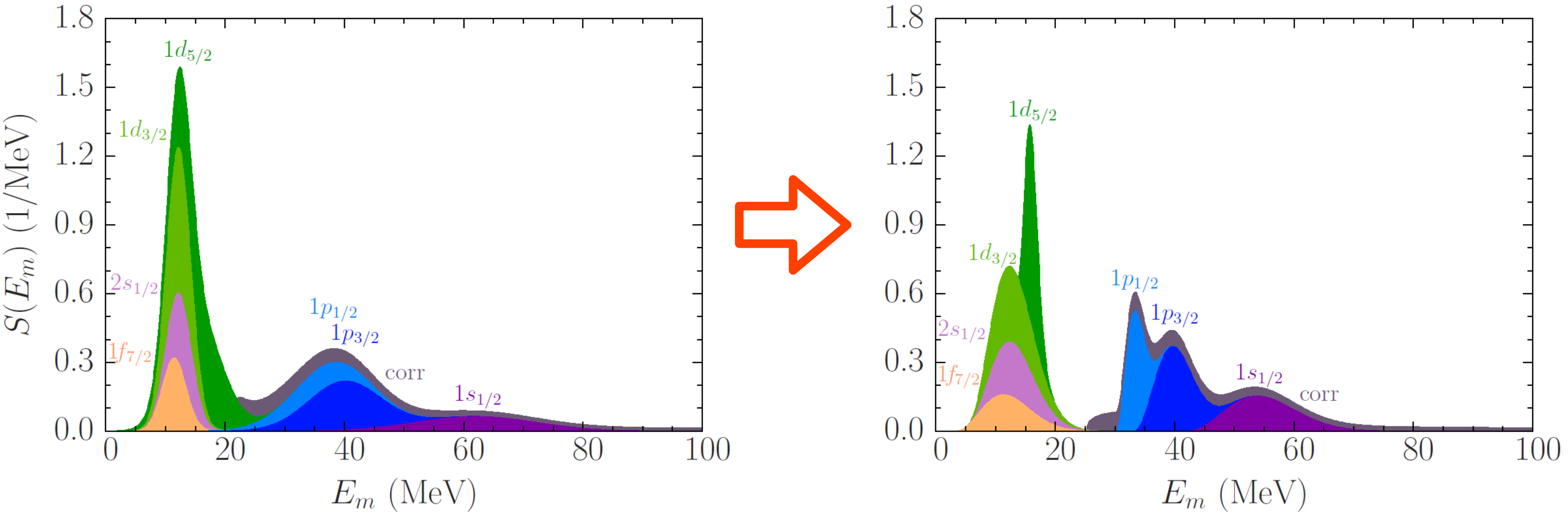
	E'_e (GeV)	θ_e (deg)	Q^2 (GeV$^2/c^2$)	$\lvert\mathbf{p}'\rvert$ (MeV/c)	$T_{p'}$ (MeV)	$\theta_{p'}$ (deg)	$\lvert\mathbf{q}\rvert$ (MeV/c)	p_m (MeV/c)	E_m (MeV)
kin1	1.777	21.5	0.549	915	372	−50.0	865	50	73
kin2	1.716	20.0	0.460	1030	455	−44.0	846	184	50
kin3	1.799	17.5	0.370	915	372	−47.0	741	174	50
kin4	1.799	15.5	0.291	915	372	−44.5	685	230	50
kin5	1.716	15.5	0.277	1030	455	−39.0	730	300	50

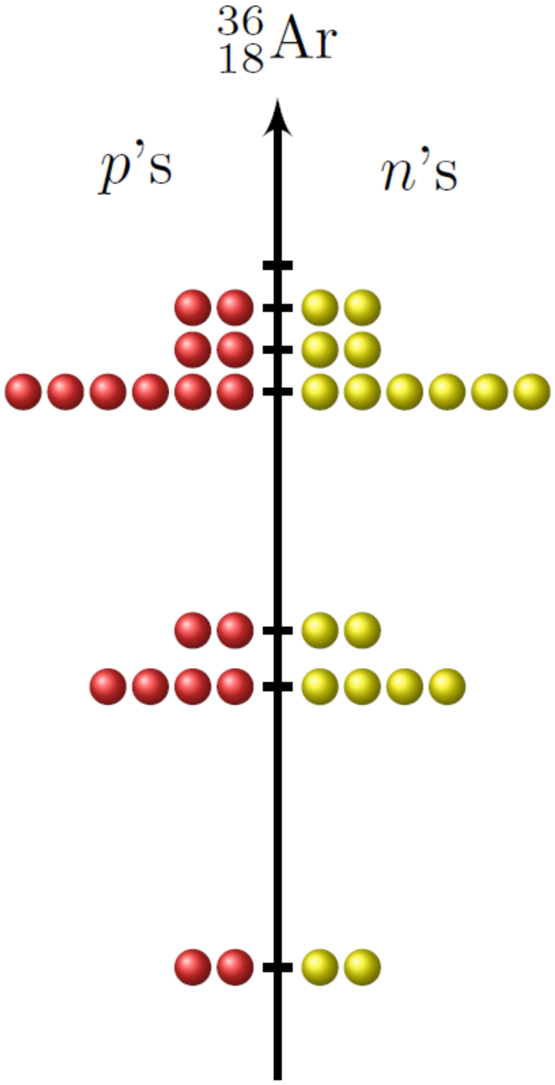


K.A. Olive *et al*. (PDG), Chin. Phys. C, 38, 090001 (2014)
https://pdg.lbl.gov/2014/hadronic-xsections/hadron.html



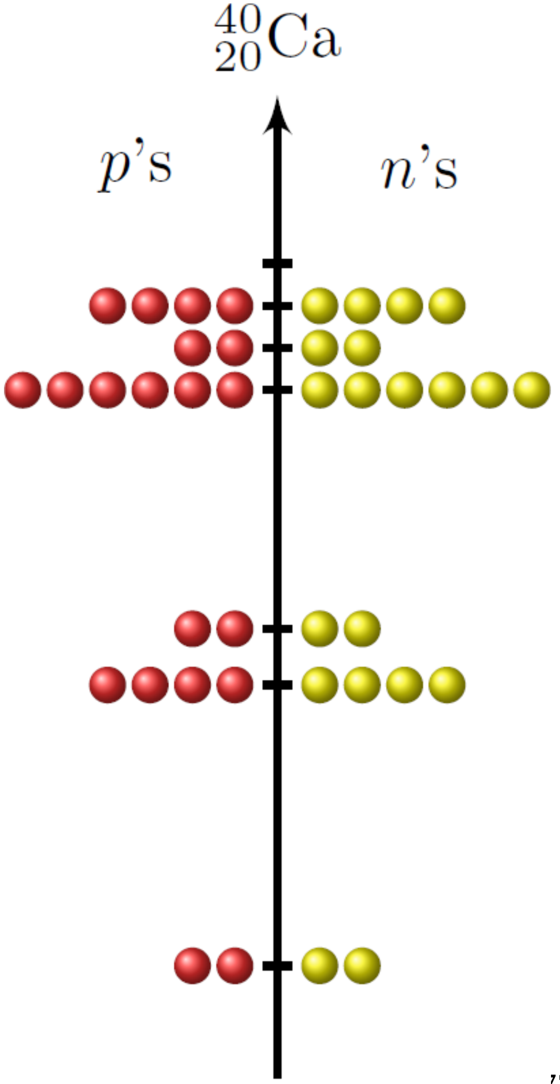
1.0
0.8
0.6
0.4
0.2
0.0
$S(E_m)$ (1/MeV)
$2s_{1/2}$
$1d_{5/2}$
$1p_{1/2}$
$1p_{3/2}$
$1d_{3/2}$
corr
$1s_{1/2}$
0
20
40
60
80
100
E_m (MeV)
1.0
0.8
0.6
0.4
0.2
0.0
$2s_{1/2}$
$1p_{1/2}$
$1p_{3/2}$
$1d_{5/2}$
$1d_{3/2}$
corr
$1s_{1/2}$
0
20
40
60
80
100
E_m (MeV)


$S(E_m)$ (1/MeV)
1.8
1.5
1.2
0.9
0.6
0.3
0.0
$1d_{5/2}$
$1d_{3/2}$
$2s_{1/2}$
$1f_{7/2}$
$1p_{1/2}$
$1p_{3/2}$
corr
$1s_{1/2}$
0
20
40
60
80
100
E_m (MeV)

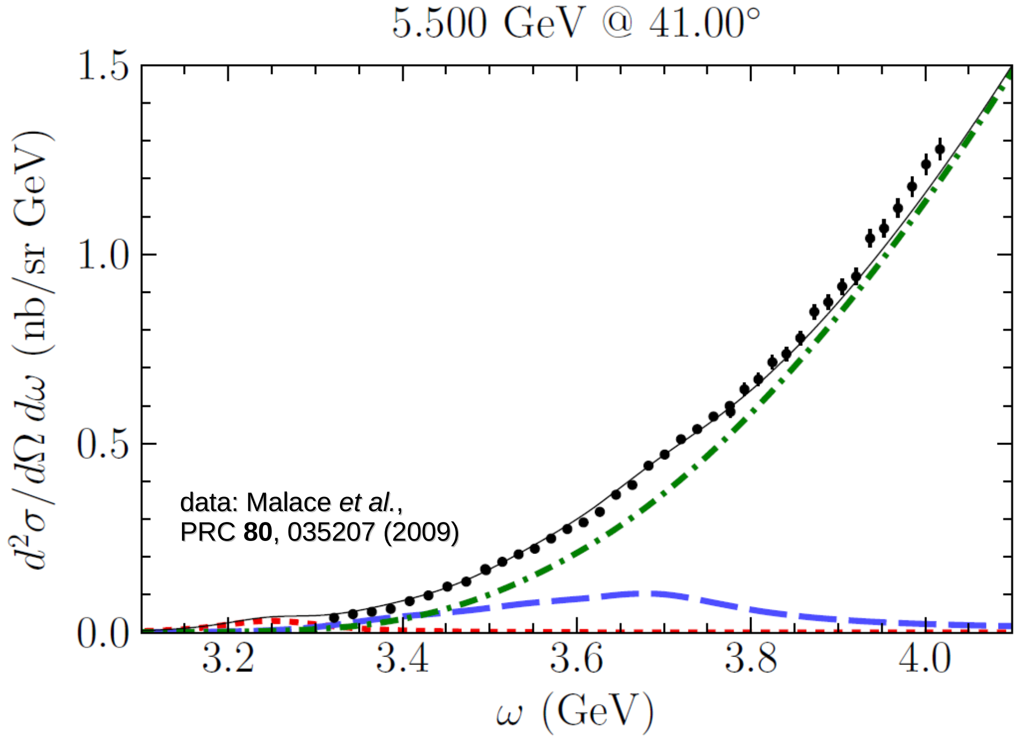


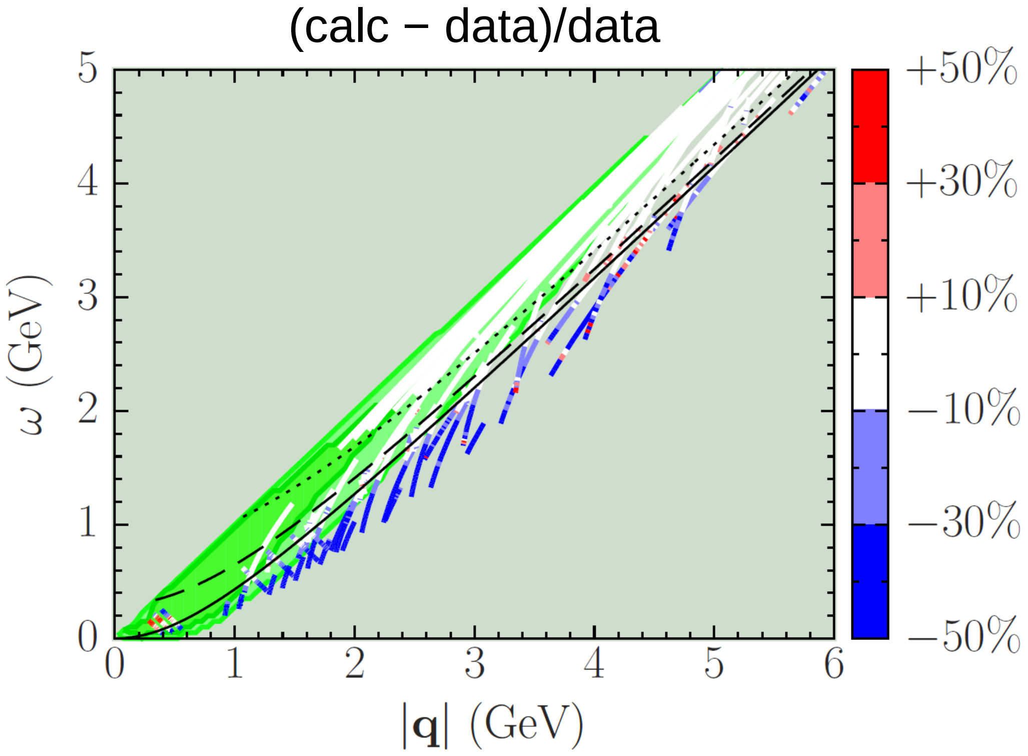
proton energy levels

Ar		Ca
8.51	1d3/2	8.33
9.73	2s1/2	10.85
14.23	1d5/2	14.66
	1p1/2	
	1p3/2	
	1s1/2	

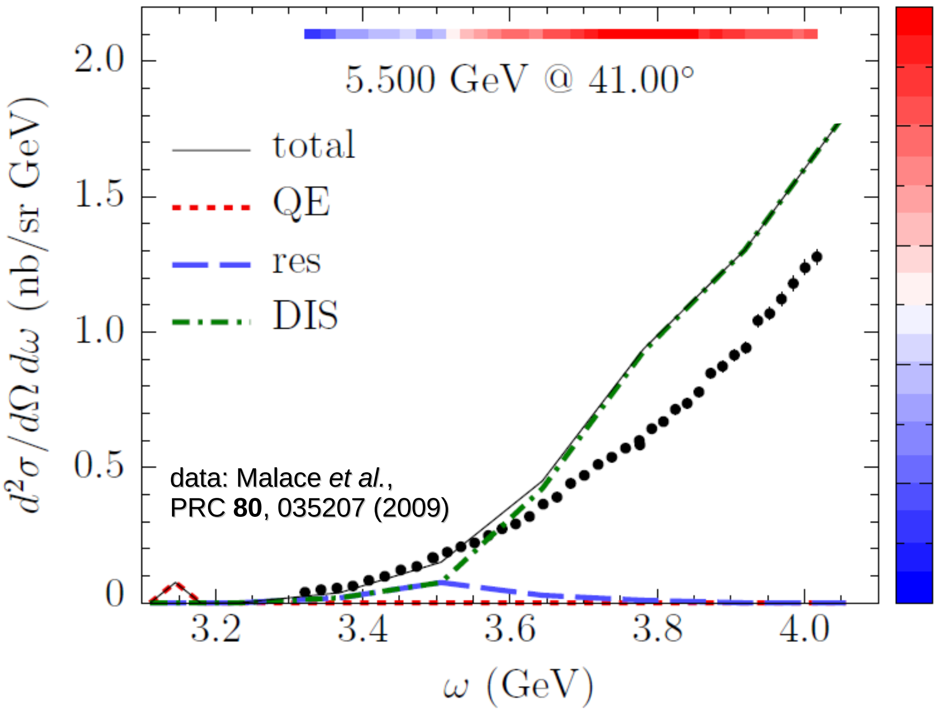


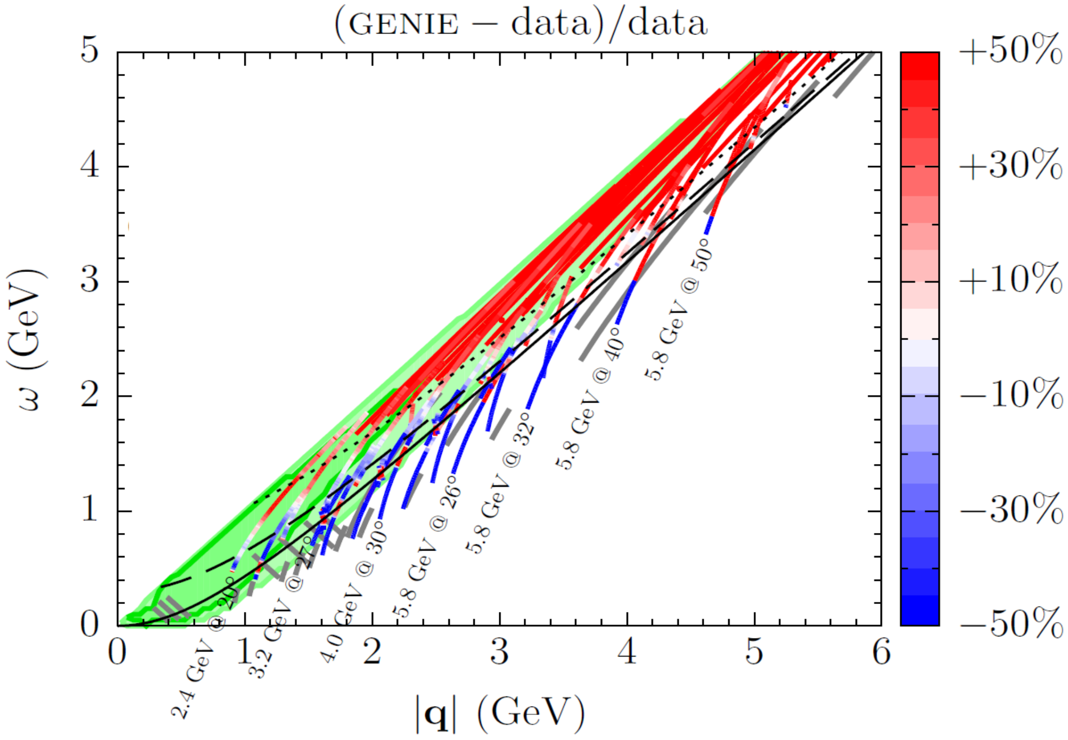
Realistic description of the nucleus: D(*e*,*e'*)





GENIE


5.500 GeV @ 41.00°
total
QE
res
DIS
data: Malace *et al.*,
PRC **80**, 035207 (2009)
$d^2\sigma/d\Omega\,d\omega$ (nb/sr GeV)
ω (GeV)


(GENIE − data)/data
ω (GeV)
$|\mathbf{q}|$ (GeV)
+50%
+30%
+10%
−10%
−30%
−50%
2.4 GeV @ 20°
3.2 GeV @ 27°
4.0 GeV @ 30°
5.8 GeV @ 26°
5.8 GeV @ 32°
5.8 GeV @ 40°
5.8 GeV @ 50°