Towards accurate neutrino cross sections: the argon and titanium spectral functions from (e,e’p) data

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based on L. Jiang et al.,
PRD 105, 112002 (2022); PRD 107, 012005 (2023)

PAC Webinar at Nanjing Normal University, May 3, 2023
The “Neutrino” by Bethe and Peierls

“The view has recently been put forward\(^1\) that a neutral particle of about electronic mass, and spin \(\frac{1}{2}\hbar\) (where \(\hbar = \frac{h}{2\pi}\)) exists, and that this ‘neutrino’ is emitted together with an electron in \(\beta\)-decay. This assumption allows the conservation laws for energy and angular momentum to hold in nuclear physics.”

“For an energy \(2.3 \times 10^6\) volts, … \(\sigma < 10^{-44}\) cm\(^2\) … With increasing energy \(\sigma\) increases … but even if one assumes a very steep increase, it seems highly improbable that, even for cosmic ray energies, \(\sigma\) becomes large enough to allow the process to be observed.”

“… one can conclude that there is no practically possible way of observing the neutrino”

\(^1\) W. Pauli, quoted repeatedly since 1931 [Dec 4, 1930], to be published shortly …, 1933 [(Gauthier-Villars, Paris, 1934)].

H. Bethe and R. Peierls, Nature 133, 532 (1934)
What did we learn since then?

- First detection—Reines and Cowan (1956, 1995)
- More than one type of neutrino exists ($\nu_{\mu} \neq \nu_e$)—Lederman, Schwartz, and Steinberger (1962, 1988)
- Direct observation of $\nu_\tau$—DONUT Collab. (2001)
Matter-antimatter asymmetry

APS/Alan Stonebraker, https://physics.aps.org/articles/v8/s17
Deep Underground Neutrino Experiment (DUNE)

- Long-baseline physics—measurement of all oscillation parameters (including the mass hierarchy and $\delta_{CP}$), test the 3-flavor framework
- Low-energy physics (supernova $\nu$’s, diffuse supernova $\nu$’s, solar $\nu$’s)
- Search for physics beyond the Standard Model
Neutrino oscillations in a nutshell

- ν's produced in a given flavor $\alpha$ (=$e$, $\mu$, $\tau$), mixture of mass eigenstates $j$ (=$1$, $2$, $3$)
- Different masses propagate with different phases, $e^{-iE_jt}$

$$tE_j = t\sqrt{p^2 + m_j^2} = t|p|\sqrt{1 + m_j^2/p^2} \approx t|p| \left(1 + \frac{m_j^2}{2p^2}\right) \approx t|p| + \frac{m_j^2 L}{2E_\nu}$$

- In the far detector, mixture of mass eigenstates is different—other flavors appear
Neutrino oscillations in a nutshell

In the simplest case of two flavors

\[ P(\nu_\alpha \rightarrow \nu_\alpha) = 1 - \sin^2 2\theta \sin^2 \left( \frac{\Delta m^2 L}{4E_{\nu}} \right) \]

In the ratio of the observed to unoscillated events,

- the position of the dip determines \( \Delta m^2 \),
- its depth determines \( \theta \)
How to make an accelerator neutrino beam

- Produce pions in proton collisions with the target.
- Focus them using horn(s).
- Let pions decay, $\pi \rightarrow \mu + \nu_\mu$, and absorb muons before they decay, $\mu \rightarrow \nu_\mu + e + \bar{\nu}_e$. 

https://news.fnal.gov/2014/12/how-to-make-a-neutrino-beam/
Neutrino energy reconstruction

Neutrino energy is converted to

- the kinetic energies of the knocked-out nucleons,
- the total energies of leptons, pions, and gammas,
- nuclear breakup
GENIE+FLUKA simulation of a 4-GeV $\nu_\mu$Ar event

Multiply differential cross sections required for energy reconstruction.
MC Generators in long-baseline neutrino physics

- Main goal: extract the $\nu$ & $\bar{\nu}$ oscillation probabilities.
- Polychromatic beams, neutrino energy reconstructed from visible energy deposited by interaction products.
- Calorimetric reconstruction of neutrino energy.
- Sizable contributions of hadrons. Neutrons’ energy estimate heavily dependent on Monte Carlo.
- Accuracy of simulations translates into the accuracy of the extracted oscillation parameters.
- We are no longer after $O(1)$ effects, without reliable cross sections precise measurements cannot succeed.
Concrete example: NOvA

"This change was caused by three changes ... The largest effect was due to new simulations and calibrations."
Concrete example: NOvA

This change was caused by three changes ... The largest effect was due to new simulations and calibrations.

Acero et al. (NOvA), PRL 118, 151802 (2017)

Acero et al. (NOvA), PRD 98, 032012 (2018)
Neutrino scattering at GeV energies

**quasielastic scattering**

\[ \nu_l + n \rightarrow l^- + p \]
\[ \bar{\nu}_l + p \rightarrow l^+ + n \]

**resonance production**

\[ \nu_l + n \rightarrow l^- + \Delta^+ \]
\[ \nu_l + p \rightarrow l^- + \Delta^{++} \]
\[ \bar{\nu}_l + n \rightarrow l^+ + \Delta^- \]
\[ \bar{\nu}_l + p \rightarrow l^+ + \Delta^0 \]
... 

**deep-inelastic scattering**

\[ \nu_l + N \rightarrow l^- + N' + n\pi \]
\[ \bar{\nu}_l + N \rightarrow l^- + N' + n\pi \]
Nuclear effects
Double differential cross section

\[ H(e, e')X \]
Double differential cross section
Double differential cross section

\[ C(e, e')X \]
Current situation

“… nuclear models available to modern neutrino experiments give similar results … none of which is confirmed by the data. … More theoretical work is needed to correctly model nuclear effects … from the quasielastic to the deep inelastic regime.”

B. G. Tice et al. (MINERvA), PRL 112, 231801 (2014)

“The double- and single-differential cross sections show similar tensions with the model predictions. These results demonstrate that improvements will need to be made to neutrino-interaction models if precision neutrino oscillation experiments hope to better constrain the systematics …”

A. Filkins et al. (MINERvA), PRD 101, 112007 (2020)
Impulse approximation

At relevant kinematics, the dominant process of neutrino-nucleus interaction is **scattering off a single nucleon**, with the remaining nucleons acting as a spectator system.

This description is valid when the momentum transfer $|q|$ is high enough ($|q| \geq 200$ MeV).
Impulse approximation

To calculate the neutrino-argon cross sections we need to know

- elementary cross sections (QE, resonant pion production, DIS ...)
- proton and neutron spectral functions (distributions of the initial momenta and energies, correlations between nucleons, ...)
- final-state interactions (nuclear transparency, optical potentials)
- hadronization
Electrons and neutrinos

For scattering in a given angle and energy, $\nu$'s and $e$'s differ almost exclusively due to the elementary cross sections.

Electron-scattering data can provide information on

- the vector contributions to elementary neutrino cross sections
- proton and neutron spectral functions (Ar & Ti targets)
- hadronization (H & D targets)
- final-state interactions (Ar & Ti + H & D targets)

Electron data allow MC validation, reduction of systematic uncertainties, as well as their rigorous determination.

Current situation

A.M.A. & Alex Friedland, PRD 102, 053001 (2020)

data: Dai et al., PRC 99, 054608 (2019)

2.222 GeV @ 15.5°
Current situation

Data: Barreau et al., NPA 402, 515 (1983)

A.M.A. & A. Friedland, PRD 102, 053001 (2020)
Fermi gas

Nucleus treated as a fragment of non-interacting infinite nuclear matter of constant density.

Eigenstates have definite momenta and energies $E_p = \sqrt{M^2 + p^2} - \epsilon$. 

![Diagram](image)
Fermi gas vs. spectral function
Realistic description of the nucleus: C(e,e’)

D(e,e’) in the Monte Carlo generator GENIE

Adopted from
A.M.A. & Alex Friedland, PRD 102, 053001 (2020)
Realistic description of $D(e,e')$ 

$$\frac{(\text{calc} - \text{data})}{\text{data}}$$

![Graph showing $d^2\sigma/d\Omega d\omega$ vs $\omega$ and $|q|$ vs $q$](image)

- **Legend**:
  - Total
  - QE
  - Res
  - DIS

- **Data**: Arrington et al., PRL 82, 2056 (1999)
E12-14-012 in JLab: \((e,e')\) and \((e,e'p)\) on Ar and Ti

**Aim:** Obtaining the experimental input indispensable to construct the argon spectral function, thus paving the way for a reliable estimate of the neutrino cross sections in DUNE. In addition, stimulating a number of theoretical developments, such as the description of final-state interactions. [Benhar et al., arXiv:1406.4080]

\[ E_e = 2.222 \text{ GeV} \]

| \( E'_e \) (GeV) | \( \theta_e \) (deg) | \( |p'| \) (MeV) | \( \theta_{p'} \) (deg) | \( |q| \) (MeV) | \( p_m \) (MeV) | \( E_m \) (MeV) |
|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|
| 1.777           | 21.5            | 915             | -50.0           | 865             | 50              | 73              |
| 1.716           | 20.0            | 1030            | -44.0           | 846             | 184             | 50              |
| 1.799           | 17.5            | 915             | -47.0           | 741             | 174             | 50              |
| 1.799           | 15.5            | 915             | -44.5           | 685             | 230             | 50              |
| 1.716           | 15.5            | 1030            | -39.0           | 730             | 300             | 50              |

First, exploratory analyses of the full datasets
Why titanium?
This analysis: extraction of the spectral function

The proton spectral function $P(p_m, E_m)$ describes the probability distribution of removing a proton of momentum $p_m$ from the target nucleus, leaving the residual system with excitation energy $E_m - E_{\text{thr}}$, with $E_{\text{thr}}$ being the proton emission threshold.
This analysis: extraction of the spectral function

Universal property of the nucleus, independent of the interaction.
Missing energy $E_m$ and missing momentum $p_m$

In general,

$$E_{*A-1} = \sqrt{(M_A - M + E_m)^2 + p_{A-1}^2}$$

$E_m - E_{thr}$ is the excitation energy of $^{39}\text{Cl}$

Without final state interactions

$$-p_{A-1} = p_m$$

is the initial proton momentum
Missing energy $E_m$ and missing momentum $p_m$

$$(E_e, k_e) \quad \quad (E'_e, k'_e)$$

$$E_p', p'$$

$E_e + M - E_m = E_e' + E_p$

known \hspace{1cm} missing

$k_e + p_m = k_e' + p'$

For negligible recoil energy,

$$E^*_A - 1 = M_A - M + E_m$$

$E_m - E_{thr}$ is the excitation energy of $^{39}\text{Cl}$

Without final state interactions

$$- p_{A-1} = p_m$$

is the initial proton momentum
Spectral function for complex nuclei

Mean-field part

- describes the shell structure
- can be determined from experimental data
- 70–80% of nucleons

Correlated part

- describes correlated nucleons
- easier to determine from theoretical estimates
Jefferson Laboratory Hall A
$^{40}\text{Ar}(e,e'p)$ in E12-14-012
\( (e,e'p) \) cross section

\[
\frac{d^4 \sigma_{IA}}{d\Omega_{k'}dE_{k'}d\Omega_{p'}dE_{p'}} \propto \sigma_{ep} S(p, E) T_A(E_{p'})
\]

T. de Forest Jr., NPA 392, 232 (1983)
Mean-field part of the spectral function

\[ P_{MF}(p_m, E_m) = \sum_\alpha S_\alpha |\phi_\alpha(p_m)|^2 f_\alpha(E_m) \]

- spectroscopic factor
- energy distribution
- wave function in momentum space

Relativistic MF calculations by C. Giusti
Mean-field part of the spectral function

<table>
<thead>
<tr>
<th>$\alpha$</th>
<th>$S_\alpha$</th>
<th>$E_\alpha$ (MeV)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$1d_{3/2}$</td>
<td>1.6</td>
<td>12.53</td>
</tr>
<tr>
<td>$2s_{1/2}$</td>
<td>1.6</td>
<td>12.93</td>
</tr>
<tr>
<td>$1d_{5/2}$</td>
<td>4.8</td>
<td>18.23</td>
</tr>
</tbody>
</table>

- $1d_{3/2}$: from the mass difference between $^{40}\text{Ar}$ and $^{39}\text{Cl} + p + e$
- $2s_{1/2}$ and $1d_{5/2}$: from the dominant contribs. in the past $^{40}\text{Ar}(d, ^3\text{He})^{39}\text{Cl}$ measurements
- Lower levels were not probed with deuteron
- Assumed Maxwell-Boltzmann distribution of missing energy

Mairle et al., NPA 565, 543 (1993)
Correlated part of the spectral function

- Correlated nucleons form quasi-deuteron pairs, with the relative momentum distributed as in deuteron.
- $NN$ pairs undergo CM motion (Gaussian distrib.)
- Excitation energy of the $(A-1)$-nucleons is their kinetic energy plus the $pn$ knockout threshold
## Missing energy distributions for Ar and Ti

<table>
<thead>
<tr>
<th>( \alpha )</th>
<th>( E_\alpha ) (MeV) w/ priors</th>
<th>( E_\alpha ) (MeV) w/o priors</th>
<th>( \sigma_\alpha ) (MeV) w/ priors</th>
<th>( \sigma_\alpha ) (MeV) w/o priors</th>
</tr>
</thead>
<tbody>
<tr>
<td>1d(_{3/2})</td>
<td>12.53 ± 0.02</td>
<td>10.90 ± 0.12</td>
<td>1.9 ± 0.4</td>
<td>1.6 ± 0.4</td>
</tr>
<tr>
<td>2s(_{1/2})</td>
<td>12.92 ± 0.02</td>
<td>12.57 ± 0.38</td>
<td>3.8 ± 0.8</td>
<td>3.0 ± 1.8</td>
</tr>
<tr>
<td>1d(_{5/2})</td>
<td>18.23 ± 0.02</td>
<td>17.77 ± 0.80</td>
<td>9.2 ± 0.9</td>
<td>9.6 ± 1.3</td>
</tr>
<tr>
<td>1p(_{1/2})</td>
<td>28.8 ± 0.7</td>
<td>28.7 ± 0.7</td>
<td>12.1 ± 1.0</td>
<td>12.0 ± 3.6</td>
</tr>
<tr>
<td>1p(_{3/2})</td>
<td>33.0 ± 0.3</td>
<td>33.0 ± 0.3</td>
<td>9.3 ± 0.5</td>
<td>9.3 ± 0.5</td>
</tr>
<tr>
<td>1s(_{1/2})</td>
<td>53.4 ± 1.1</td>
<td>53.4 ± 1.0</td>
<td>28.3 ± 2.2</td>
<td>28.1 ± 2.3</td>
</tr>
<tr>
<td>corr.</td>
<td>24.1 ± 2.7</td>
<td>24.1 ± 1.7</td>
<td>—</td>
<td>—</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>( \alpha )</th>
<th>( E_\alpha ) (MeV) w/ priors</th>
<th>( E_\alpha ) (MeV) w/o priors</th>
<th>( \sigma_\alpha ) (MeV) w/ priors</th>
<th>( \sigma_\alpha ) (MeV) w/o priors</th>
</tr>
</thead>
<tbody>
<tr>
<td>1f(_{7/2})</td>
<td>11.32 ± 0.10</td>
<td>11.31 ± 0.10</td>
<td>8.00 ± 5.57</td>
<td>8.00 ± 6.50</td>
</tr>
<tr>
<td>1d(_{3/2})</td>
<td>12.30 ± 0.24</td>
<td>12.33 ± 0.24</td>
<td>7.00 ± 0.61</td>
<td>7.00 ± 3.84</td>
</tr>
<tr>
<td>2s(_{1/2})</td>
<td>12.77 ± 0.25</td>
<td>12.76 ± 0.25</td>
<td>7.00 ± 3.76</td>
<td>7.00 ± 3.84</td>
</tr>
<tr>
<td>1d(_{5/2})</td>
<td>15.86 ± 0.20</td>
<td>15.91 ± 0.22</td>
<td>2.17 ± 0.27</td>
<td>2.23 ± 0.29</td>
</tr>
<tr>
<td>1p(_{1/2})</td>
<td>33.33 ± 0.60</td>
<td>33.15 ± 0.65</td>
<td>3.17 ± 0.45</td>
<td>3.03 ± 0.48</td>
</tr>
<tr>
<td>1p(_{3/2})</td>
<td>39.69 ± 0.62</td>
<td>39.43 ± 0.68</td>
<td>5.52 ± 0.70</td>
<td>5.59 ± 0.70</td>
</tr>
<tr>
<td>1s(_{1/2})</td>
<td>53.84 ± 1.86</td>
<td>52.00 ± 3.13</td>
<td>11.63 ± 1.90</td>
<td>13.63 ± 2.59</td>
</tr>
<tr>
<td>corr.</td>
<td>25.20 ± 0.02</td>
<td>25.00 ± 0.29</td>
<td>—</td>
<td>—</td>
</tr>
</tbody>
</table>
Spectroscopic factors for Ar and Ti

<table>
<thead>
<tr>
<th>$\alpha$</th>
<th>$N_\alpha$</th>
<th>all priors</th>
<th>w/o $p_m$</th>
<th>w/o corr.</th>
</tr>
</thead>
<tbody>
<tr>
<td>$1d_{3/2}$</td>
<td>2</td>
<td>$0.89 \pm 0.11$</td>
<td>$1.42 \pm 0.20$</td>
<td>$0.95 \pm 0.11$</td>
</tr>
<tr>
<td>$2s_{1/2}$</td>
<td>2</td>
<td>$1.72 \pm 0.15$</td>
<td>$1.22 \pm 0.12$</td>
<td>$1.80 \pm 0.16$</td>
</tr>
<tr>
<td>$1d_{5/2}$</td>
<td>6</td>
<td>$3.52 \pm 0.26$</td>
<td>$3.83 \pm 0.30$</td>
<td>$3.89 \pm 0.30$</td>
</tr>
<tr>
<td>$1p_{1/2}$</td>
<td>2</td>
<td>$1.53 \pm 0.21$</td>
<td>$2.01 \pm 0.22$</td>
<td>$1.83 \pm 0.21$</td>
</tr>
<tr>
<td>$1p_{3/2}$</td>
<td>4</td>
<td>$3.07 \pm 0.05$</td>
<td>$2.23 \pm 0.12$</td>
<td>$3.12 \pm 0.05$</td>
</tr>
<tr>
<td>$1s_{1/2}$</td>
<td>2</td>
<td>$2.51 \pm 0.05$</td>
<td>$2.05 \pm 0.23$</td>
<td>$2.52 \pm 0.05$</td>
</tr>
<tr>
<td>corr.</td>
<td>0</td>
<td>$3.77 \pm 0.28$</td>
<td>$3.85 \pm 0.25$</td>
<td>excluded</td>
</tr>
<tr>
<td>$\sum_\alpha S_\alpha$</td>
<td>17.02 ± 0.48</td>
<td>16.61 ± 0.57</td>
<td>14.12 ± 0.42</td>
<td></td>
</tr>
<tr>
<td>d.o.f</td>
<td>206</td>
<td>231</td>
<td>232</td>
<td></td>
</tr>
<tr>
<td>$\chi^2$/d.o.f.</td>
<td>1.9</td>
<td>1.4</td>
<td>2.0</td>
<td></td>
</tr>
</tbody>
</table>

| $1f_{7/2}$ | 2          | $1.53 \pm 0.25$ | $1.55 \pm 0.28$ | $1.24 \pm 0.22$ |
| $1d_{3/2}$ | 4          | $2.79 \pm 0.37$ | $3.15 \pm 0.54$ | $3.21 \pm 0.37$ |
| $2s_{1/2}$ | 2          | $2.00 \pm 0.11$ | $1.78 \pm 0.46$ | $2.03 \pm 0.11$ |
| $1d_{5/2}$ | 6          | $2.25 \pm 0.16$ | $2.34 \pm 0.19$ | $3.57 \pm 0.29$ |
| $1p_{1/2}$ | 2          | $2.00 \pm 0.20$ | $1.80 \pm 0.27$ | $2.09 \pm 0.19$ |
| $1p_{3/2}$ | 4          | $2.90 \pm 0.20$ | $2.92 \pm 0.20$ | $4.07 \pm 0.15$ |
| $1s_{1/2}$ | 2          | $2.14 \pm 0.10$ | $2.56 \pm 0.30$ | $2.14 \pm 0.11$ |
| corr.     | 0          | $4.71 \pm 0.31$ | $4.21 \pm 0.46$ | excluded  |
| $\sum_\alpha S_\alpha$ | 20.32 ± 0.65 | 20.30 ± 1.03 | 18.33 ± 0.59 |
| d.o.f     | 121        | 153          | 125         |
| $\chi^2$/d.o.f. | 0.95         | 0.71          | 1.23         |
Partial momentum distributions

Data from different kinematics are consistent within uncertainties.
Energy levels

\[
\begin{array}{ccc}
40\text{Ar} & 48\text{Ti} \\
n\text{e} & p\text{e} \\
\text{neutrons} & \text{protons} \\
9.87 & 1f7/2 & 11.45 \\
11.39 & 1d3/2 & 12.21 \\
12.23 & 2s1/2 & 12.84 \\
13.23 & 1d5/2 & 15.45 \\
\end{array}
\]

Agreement to 0.6–2.2 MeV
52-MeV polarized [Doll et al., JPG 5, 1421 (1979); $E_x < 7.54$ MeV] deuteron beam at Karlsruhe
Occupation probability

52-MeV polarized [Mairle et al., NPA 565, 543 (1993); $E_x < 9$ MeV] and unpolarized [Doll et al., NPA 230, 329 (1974); 129, 469 (1969); $E_x < 7$ MeV] deuteron beam at Karlsruhe

Kramer et al. [NPA 679, 267 (2001)]: reanalysis of $(d,^3\text{He})$ experiments, $S_\alpha \rightarrow S_\alpha/1.5$
### Proton Energy Levels

<table>
<thead>
<tr>
<th>Element</th>
<th>1s1/2</th>
<th>1p1/2</th>
<th>1p3/2</th>
<th>1d3/2</th>
<th>1d5/2</th>
<th>2s1/2</th>
<th>1d5/2</th>
<th>1d3/2</th>
<th>2s1/2</th>
<th>1d5/2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ar</td>
<td>53.4(1.1)</td>
<td>28.8(7)</td>
<td>33.0(3)</td>
<td>12.53(2)</td>
<td>18.23(2)</td>
<td>12.92(2)</td>
<td>12.92(2)</td>
<td>12.53(2)</td>
<td>12.92(2)</td>
<td>18.23(2)</td>
</tr>
<tr>
<td>Ca</td>
<td>53.6(7)</td>
<td>29.8(7)</td>
<td>34.7(3)</td>
<td>8.5(1)</td>
<td>11.0(1)</td>
<td>15.7(1)</td>
<td>15.7(1)</td>
<td>11.0(1)</td>
<td>15.7(1)</td>
<td>15.7(1)</td>
</tr>
</tbody>
</table>

- **Ar**: 12.53(2) $^1d_{3/2}$, 12.92(2) $^2s_{1/2}$, 18.23(2) $^1d_{5/2}$
- **Ca**: 8.5(1) $^1s_{1/2}$, 11.0(1) $^2s_{1/2}$, 15.7(1) $^1d_{5/2}$

*Jiang et al., PRD 105, 112002 (2022)*  
*Volkov et al., SJNP 52, 848 (1990)*
Occupation probability


Yasuda et al. [Ph.D. thesis (2012)]: 392-MeV polarized proton beam at RCNP
Directions for future improvements

- 2D analysis
- Final-state interactions
- Wave functions
- Correlated part of the spectral function
Summary

• The success of the long-baseline neutrino program requires reliable cross sections.
• The spectral function approach is a viable option.
• The first, exploratory analysis of the full dataset of the JLab experiment E12-14-012 found reasonable parametrizations of the spectral functions of $^{40}$Ar and $^{48}$Ti.
• Comparison with past results shows strengths and limitations.
• Separation of individual contributions requires improved analysis. Numerous theoretical developments are necessary.
Concrete example: NOvA

Acero et al. (NOvA), PRL 118, 151802 (2017)

Acero et al. (NOvA), PRD 98, 032012 (2018)

“This change was caused by three changes ... The largest effect was due to new simulations and calibrations.”
Concrete example: NOvA

Acero et al. (NOvA), PRL 118, 151802 (2017)

"This change was caused by three changes ... The largest effect was due to new simulations and calibrations."
Neutrino double differential cross section

$\theta_\mu = 15^\circ$

$E_\nu = 3.34$ GeV

(average DUNE energy for the 2016 flux)

A.M.A. & A. Friedland, PRD 102, 053001 (2020)
Neutrino double differential cross section

\[ \theta_\mu = 15^\circ \]

\[ d^2\sigma/d\Omega dE_\mu \] (10^{-38} \text{ cm}^2/\text{sr GeV})

A.M.A. & A. Friedland, PRD 102, 053001 (2020)
Previous results

- Inclusive cross sections for C and Ti [Dai et al., PRC 98, 014617 (2018)]
- Inclusive cross section for Ar [Dai et al., PRC 99, 054608 (2019)]
- Inclusive cross section for Al-7075, A-, y-,ψ-scaling of all \((e,e')\) data [Murphy et al., PRC 100, 054606 (2019)]
- Exclusive Ar & Ti cross sections for a single kinematics, \(p_m \sim 50–60\) MeV, \(E_m \sim 50–70\) MeV [Gu et al., PRC 103, 034604 (2021)]
Partial momentum distributions

Data from different kinematics are consistent within uncertainties.
Test spectral function

Extracted spectral function
In the $p_m$ fit, only deeply bound states are sensitive to the correlated spectral function.
$E_m$ fit results for Ti

<table>
<thead>
<tr>
<th>$\alpha$</th>
<th>$N_\alpha$</th>
<th>$S_{\alpha}$</th>
<th>w/o $p_m$</th>
<th>w/o corr.</th>
</tr>
</thead>
<tbody>
<tr>
<td>$1f_{7/2}$</td>
<td>2</td>
<td>1.53 ± 0.25</td>
<td>1.55 ± 0.28</td>
<td>1.24 ± 0.22</td>
</tr>
<tr>
<td>$1d_{3/2}$</td>
<td>4</td>
<td>2.79 ± 0.37</td>
<td>3.15 ± 0.54</td>
<td>3.21 ± 0.37</td>
</tr>
<tr>
<td>$2s_{1/2}$</td>
<td>2</td>
<td>2.00 ± 0.11</td>
<td>1.78 ± 0.46</td>
<td>2.03 ± 0.11</td>
</tr>
<tr>
<td>$1d_{5/2}$</td>
<td>6</td>
<td>2.25 ± 0.16</td>
<td>2.34 ± 0.19</td>
<td>3.57 ± 0.29</td>
</tr>
<tr>
<td>$1p_{1/2}$</td>
<td>2</td>
<td>2.00 ± 0.20</td>
<td>1.80 ± 0.27</td>
<td>2.09 ± 0.19</td>
</tr>
<tr>
<td>$1p_{3/2}$</td>
<td>4</td>
<td>2.90 ± 0.20</td>
<td>2.92 ± 0.20</td>
<td>4.07 ± 0.15</td>
</tr>
<tr>
<td>$1s_{1/2}$</td>
<td>2</td>
<td>2.14 ± 0.10</td>
<td>2.56 ± 0.30</td>
<td>2.14 ± 0.11</td>
</tr>
<tr>
<td>corr.</td>
<td>0</td>
<td>4.71 ± 0.31</td>
<td>4.21 ± 0.46</td>
<td>excluded</td>
</tr>
</tbody>
</table>

$\sum_\alpha S_{\alpha} = 20.32 ± 0.65$  
$\text{d.o.f.} = 121$,  
$\chi^2/\text{d.o.f.} = 0.95$.

\[ 130 < p_m < 260 \text{ MeV} \]

\[ E_\alpha \text{ (MeV)} \]

\[ \sigma_\alpha \text{ (MeV)} \]

<table>
<thead>
<tr>
<th>$\alpha$</th>
<th>w/ priors</th>
<th>w/o priors</th>
<th>w/ priors</th>
<th>w/o priors</th>
</tr>
</thead>
<tbody>
<tr>
<td>$1f_{7/2}$</td>
<td>11.32 ± 0.10</td>
<td>11.31 ± 0.10</td>
<td>8.00 ± 5.57</td>
<td>8.00 ± 6.50</td>
</tr>
<tr>
<td>$1d_{3/2}$</td>
<td>12.30 ± 0.24</td>
<td>12.33 ± 0.24</td>
<td>7.00 ± 0.61</td>
<td>7.00 ± 3.84</td>
</tr>
<tr>
<td>$2s_{1/2}$</td>
<td>12.77 ± 0.25</td>
<td>12.76 ± 0.25</td>
<td>7.00 ± 3.76</td>
<td>7.00 ± 3.84</td>
</tr>
<tr>
<td>$1d_{5/2}$</td>
<td>15.86 ± 0.20</td>
<td>15.91 ± 0.22</td>
<td>2.17 ± 0.27</td>
<td>2.23 ± 0.29</td>
</tr>
<tr>
<td>$1p_{1/2}$</td>
<td>33.33 ± 0.60</td>
<td>33.15 ± 0.65</td>
<td>3.17 ± 0.45</td>
<td>3.03 ± 0.48</td>
</tr>
<tr>
<td>$1p_{3/2}$</td>
<td>39.69 ± 0.62</td>
<td>39.43 ± 0.68</td>
<td>5.52 ± 0.70</td>
<td>5.59 ± 0.70</td>
</tr>
<tr>
<td>$1s_{1/2}$</td>
<td>53.84 ± 1.86</td>
<td>52.00 ± 3.13</td>
<td>11.63 ± 1.90</td>
<td>13.63 ± 2.59</td>
</tr>
<tr>
<td>corr.</td>
<td>25.20 ± 0.02</td>
<td>25.00 ± 0.29</td>
<td>—</td>
<td>—</td>
</tr>
</tbody>
</table>

\[ E_m \text{ (MeV)} \]
# Proton Energy Levels

The proton energy levels of $^{40}_{18}$Ar and $^{48}_{22}$Ti are shown in the diagrams. The energy levels are listed in the table below:

<table>
<thead>
<tr>
<th></th>
<th>$^{40}_{18}$Ar</th>
<th>$^{48}_{22}$Ti</th>
</tr>
</thead>
<tbody>
<tr>
<td>1f7/2</td>
<td>11.32(10)</td>
<td>11.32(10)</td>
</tr>
<tr>
<td>1d3/2</td>
<td>12.30(24)</td>
<td>12.30(24)</td>
</tr>
<tr>
<td>2s1/2</td>
<td>12.77(25)</td>
<td>12.77(25)</td>
</tr>
<tr>
<td>1d5/2</td>
<td>15.86(20)</td>
<td>15.86(20)</td>
</tr>
<tr>
<td>1p1/2</td>
<td>33.3(6)</td>
<td>33.3(6)</td>
</tr>
<tr>
<td>1p3/2</td>
<td>39.7(6)</td>
<td>39.7(6)</td>
</tr>
<tr>
<td>1s1/2</td>
<td>53.8(1.9)</td>
<td>53.8(1.9)</td>
</tr>
</tbody>
</table>

The diagrams illustrate the distribution of protons and neutrons in the respective isotopes. The energy levels are indicated with different colors and states.

The data is from Jiang et al., PRD 105, 112002 (2022) and Jiang et al., PRD 107, 012005 (2023).
Calcium isotopes

<table>
<thead>
<tr>
<th></th>
<th>40Ca</th>
<th>48Ca</th>
</tr>
</thead>
<tbody>
<tr>
<td>8.3(3)</td>
<td>1d3/2</td>
<td>16.8(3)</td>
</tr>
<tr>
<td>11.1(3)</td>
<td>2s1/2</td>
<td>17.1(3)</td>
</tr>
<tr>
<td>16.8(4)</td>
<td>1d5/2</td>
<td>23.9(7)</td>
</tr>
</tbody>
</table>


6–8.5 MeV differences
|             | $E'_c$   | $\theta_c$ | $Q^2$     | $|p'|$    | $T_{\nu'}$ | $\theta_{\nu'}$ | $|q|$    | $p_{\nu}$ | $E_{\nu}$ |
|-------------|----------|------------|-----------|----------|------------|------------------|----------|-----------|-----------|
|             | (GeV)    | (deg)      | (GeV$^2$/c$^2$) | (MeV/c) | (MeV)      | (deg)          | (MeV/c)  | (MeV/c)   | (MeV)     |
| kin1        | 1.777    | 21.5       | 0.549     | 915      | 372        | -50.0           | 865      | 50        | 73        |
| kin2        | 1.716    | 20.0       | 0.460     | 1030     | 455        | -44.0           | 846      | 184       | 50        |
| kin3        | 1.799    | 17.5       | 0.370     | 915      | 372        | -47.0           | 741      | 174       | 50        |
| kin4        | 1.799    | 15.5       | 0.291     | 915      | 372        | -44.5           | 685      | 230       | 50        |
| kin5        | 1.716    | 15.5       | 0.277     | 1030     | 455        | -39.0           | 730      | 300       | 50        |
https://pdg.lbl.gov/2014/hadronic-xsections/hadron.html
Proton energy levels

<table>
<thead>
<tr>
<th>Element</th>
<th>Ar</th>
<th>Ca</th>
</tr>
</thead>
<tbody>
<tr>
<td>1s1/2</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1p1/2</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1p3/2</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1d3/2</td>
<td>8.51</td>
<td>8.33</td>
</tr>
<tr>
<td>2s1/2</td>
<td>9.73</td>
<td>10.85</td>
</tr>
<tr>
<td>1d5/2</td>
<td>14.23</td>
<td>14.66</td>
</tr>
</tbody>
</table>
Realistic description of the nucleus: \( D(e,e') \)

\[
\frac{d^3 \sigma}{d\Omega \ d\omega} \text{ (mb/sr GeV)}
\]

\[
\omega \text{ (GeV)}
\]

\[
5.500 \text{ GeV @ 41.00°}
\]

\[
\text{(calc – data)/data}
\]

\[
|q| \text{ (GeV)}
\]

\[
\text{data: Malace et al., PRC 80, 035207 (2009)}
\]
GENIE

\[ \frac{d^2\sigma}{d\Omega d\omega} \text{ (nb/sr GeV)} \]

- total
- QE
- res
- DIS

\[ \omega \text{ (GeV)} \]

\[ |q| \text{ (GeV)} \]

data: Malace et al., PRC 80, 035207 (2009)